Error Analysis of Multi-Hop Free-Space Optical Communication

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ABSTRACT

In this paper we analyze the error performance of Free-Space Optical (FSO) communication over multiple hops. We first develop an error model for a single hop based on visibility, atmospheric attenuation, and geometric spread of the light beam. We model atmospheric visibility by Gaussian distributions with mean and variance values to reflect clear and adverse weather conditions. Based on this, we find the end-to-end bit error distribution of the FSO link for single hop and multi-hop scenarios.

We present simulation results for decoded relaying, where each hop decodes the signal before retransmitting. We demonstrate that multi-hop FSO communication achieves a significant reduction in the mean bit error rate and also reduces the variance of the bit error rate. We argue that by lowering mean error and error variance, multi-hop operation facilitates an efficient system design and improves the reliability of the FSO link by application of specific coding schemes (such as Forward Error Correction techniques).

I. INTRODUCTION

Optical wireless, also known as free-space optics (FSO), is an effective high bandwidth communication technology serving commercial point-to-point links in terrestrial last mile applications and in infrared indoor LANs. Free space being the medium for signal transmission, FSO links suffer from reduced reliability during adverse atmospheric conditions. To increase the reliability of an FSO link, two important methods have been proposed in the literature [1], [2]. One is to provide hybrid link protection using an RF link, and the other is scaling the hop length down between the transmitter and receiver using multi-hop routing. This paper focuses on the second approach, increasing the FSO link reliability by using smaller, multiple hops. The objective of the paper is to understand the benefits and limitations of the multi-hop approach for FSO networks.

We present the error behavior due to atmospheric and geometric attenuation of the FSO signal for both single hop and multiple hop cases. We show that multiple hops enhance the reliability of the FSO link in both clear weather and bad weather conditions by reducing the mean and variance of end-to-end error. Since the mean and the variance of the error is reduced, we can design efficient error control codes to operate with FSO links. With this approach, we argue that FSO links can be made sufficiently reliable to be considered for last mile and metropolitan networks.

We model the multi-hop FSO communication system with a source terminal and a receiving terminal at the two ends, and a fixed number of intermediate relaying terminals. Each of the intermediate relaying terminals may either have the ability to decode the received signal or just amplify it before retransmitting. Since the FSO channel is slowly varying, we assume the attenuation experienced by a single bit to be a constant during its transit. The attenuation and error behavior for the individual hops is assumed to be independent.

We model the atmospheric visibility as a Gaussian random variable. We find the end-to-end error distribution for single hop and multiple hop cases, taking into account the effect of hop length and number of hops. Visibility for the clear weather and bad weather cases is modelled using different mean and variance values.

Errors on an FSO link can be modeled as *random errors* caused by attenuation and reduced signal-to-noise ratio (SNR) due to bad visibility conditions like rain, snow, and fog, and *burst errors* due to occasional obstructions and cloud-bursts. In this paper we discuss the random errors caused by attenuation on the FSO channel. During clear weather conditions the FSO link has very low Bit Error Rate (BER), almost acting as a wired link. However, during adverse weather conditions, the BER due to random errors can be very high due to drastically reduced SNR. We propose two design metrics to evaluate the performance enhancements due to multiple hops. The first metric measures the reduction in the magnitude of the average BER by the use of multiple hops. The second metric captures the reduction in the variance of the error in the multiple hop case compared to single hop.

The rest of the paper is organized as follows: In the next section we introduce a single hop FSO system and its components. In Section III we present the error behavior of an FSO link over a single hop. In Section IV, we use simulation to analyze the error accumulation and distribution over multiple hops, and compare it with single hop with decoded relaying. In Section V we briefly introduce multi-hop systems with only amplified relaying. We conclude the paper in Section VI with directions for future work.

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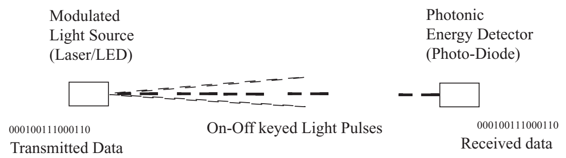

Fig. 1. Typical single hop FSO communication system.

II. Single Hop FSO System

A single hop FSO communication system is shown in Figure 1. The transmitter is a modulated light source, typically a low powered laser operating in infrared band. The receiver is a photodetector, outputs a current proportional to the received light intensity. The distance between the transmitter and the receiver is the range of the FSO link. A typical FSO link is duplex, consisting of a transmitter and a receiver at both the ends of communication.

The modulation scheme used in FSO communications is On–Off keying. It is a digital modulation method where a carrier is switched ON to transmit a ONE and switched OFF to transmit a ZERO. In the case of FSO communication, a light beam from a laser acts as the carrier.

Over a single hop, the output signal at the receiver can be written as

$$y = a(t)x + n$$

where, y is the signal received and $a(t)$ is the attenuation as a function of time t, experienced by the input signal x. n is the additive white Gaussian noise (AWGN) caused by the receiver circuit [3].

The transmitted signal x is subjected to attenuation due to geometric spread, and the suspended particles in the atmosphere at various weather conditions [4]. The attenuation experienced by the signal causes random errors at the receiver due to reduced Signal to Noise Ratio (SNR). Turbulence in the atmosphere also causes errors due to distortion in the signal, which is not discussed in this paper. The total attenuation due to atmospheric propagation and geometric spread can be expressed as:

$$a(t) = a_G \cdot a_A(t)$$

where a_G is the attenuation due to geometric spread and a_A is the attenuation due to atmospheric propagation.

In an FSO communication system, the geometric spread is a fixed function of the specific design components of the system and is given by [4]:

$$a_G = \frac{SA_R}{SA_T + \frac{\pi}{4}(\theta R)^2}$$

where SA_R is the area of the receiver, SA_T is the area of the transmitter, θ is the angular divergence of the light source, and R is the distance between the transmitter and the receiver.

The atmospheric attenuation is a time varying factor, which depends essentially on the visibility between the sender and receiver at the instant when the packet is being transmitted. It is given by [4]:

$$a_A(t) = e^{-\rho R}$$

where

$$\rho = \frac{3.91}{V(t)}\left(\frac{\lambda}{550nm}\right)^{-q}$$

where $V(t)$ is the atmospheric visibility at a given time t, λ is the wavelegth of the optical signal used, and q is the size of the suspended particles in the signal transmission path [4].

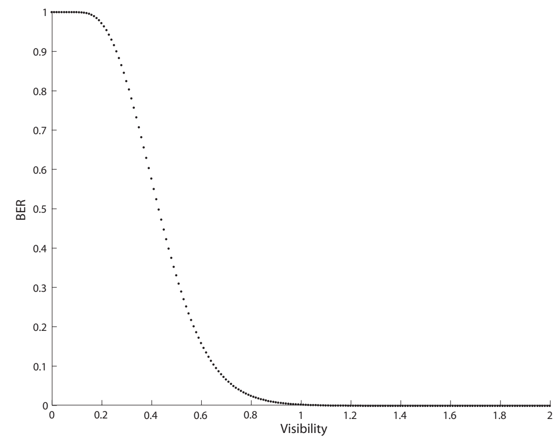

Fig. 2. BER variation per hop with visibility.

III. Error Analysis of a Single hop FSO channel

A single hop FSO link can be modelled as a Binary Symmetric Channel (BSC) with an error probability P_e. The probability of error for such a channel with on-off keying is given by [5]:

$$P_e = Q(a(t)\sqrt{SNR})$$

where Q is the error function. Since the attenuation $a(t)$ is a function of the visibility, P_e, and hence the BER is a function of visibility. Since we assume a gaussian model for atmospheric visibility we obtain the distribution of P_e and hence for BER for each hop as shown in Figure 2.

Figure 3 illustrates the variation of the BER with SNR for different visibilites. As the visibility becomes worse, the received SNR decreases and the probability of error increases. An FSO system designed to work with a single visibility, and hence a fixed received SNR performs worse as the weather degrades. The channel behavior is worst for foggy conditions in the case of FSO communications.

The strategy to combat the degrading behavior due to decreasing visibility is either to have an adaptive strategy to increase the transmitted power keeping the SNR fixed as the weather degrades, or always leave a fixed power margin so as to work for a broad range of visibilities. The first method even though is more energy efficient than the second, is hard to achieve in reality, as it demands the channel state information time to time. We propose the use of multiple hops

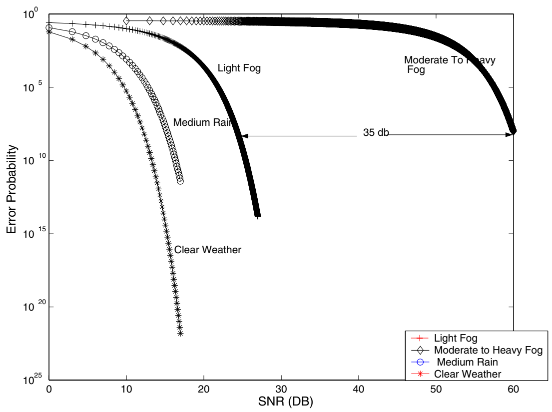

Fig. 3. Error probability over a single hop with SNR for different visibilities.

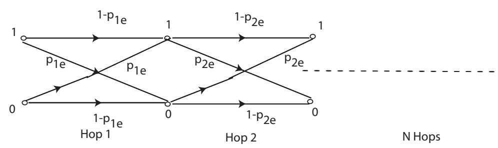

Fig. 4. Multi-hop equivalent channel model.

to minimize the error and also reduce its variance. By reducing the variance, we can design an FSO system that can be reliable for a wide range of channel conditions more efficiently.

IV. Multi-Hop System: Decoded Relaying

The multi-hop FSO channel with N hops is modelled as a concatenation of N BSCs. It is illustrated in Figure 4. Since each channel is assumed to be independent, the end-to-end error probability and hence the BER for the multi-hop channel over N hops is given by:

$$P_{e(multi-hop)} = 1 - ((1 - p_1)(1 - p_2)....(1 - p_N))$$

Assuming that each of the crossover error probabilities of the BSC are on the order of 10^{-2}, the above expression is approximated for the clear weather conditions as:

$$P_{e(multi-hop)} = \Sigma_i^N p_i$$

We use this approximation for the simulation of end-to-end error accumulation over multiple hops in clear weather conditions.

In the case of decoded relaying, the multi-hop channel corresponds to the case where each intermediate terminal decodes the received signal and re-encodes before retransmission. This system does not propagate noise, as at each hop, the signal is reconstructed with a finite decoding error. At each stage there is also a delay which is accumulated over the total number of hops. For a given end-to-end length, the system can be operated as a single hop, or can be divided into multiple hops. The relationship between the number of hops and the error rate

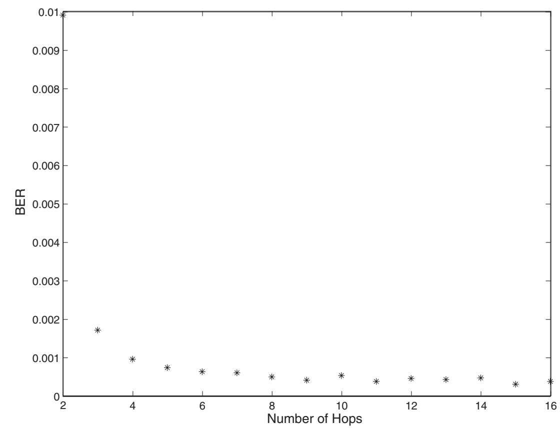

Fig. 5. BER versus number of hops for a fixed link length.

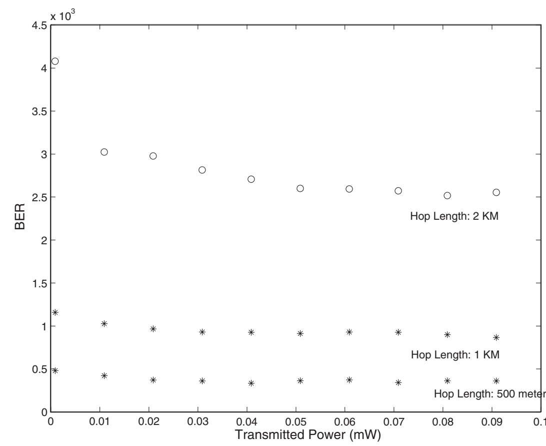

Fig. 6. Transmitted power versus hoplength.

helps to determine how many hops have to be implemented. From Figure 5 we can get an estimate of how many hops are optimal for desired operation. As seen, the decrease in the error is not significant after 8 hops for the given visibility distribution and the end-to-end link length.

The effect of hoplength on the transmitted power and the resulting error rate can be seen from Figure 6. For the same transmitted power, the resulting error rate decreases as the hoplength decreases.

Figure 7 shows how error gets accumulated over multiple hops as the hoplength is increased. The error remains low till a hop length value of 500 meters and starts to build rapidly after that. Using this result, we fixed our hoplength at 500 meters to simulate multi-hop error behavior.

In the next two subsections, we find the error distribution over single hop and multi-hop scenarios for clear weather conditions and adverse weather conditions.

A. Clear Weather Conditions

For the case of clear weather, the visibility is taken as a Gaussian with a mean at 10 KMs and a variance of 3 Kms,

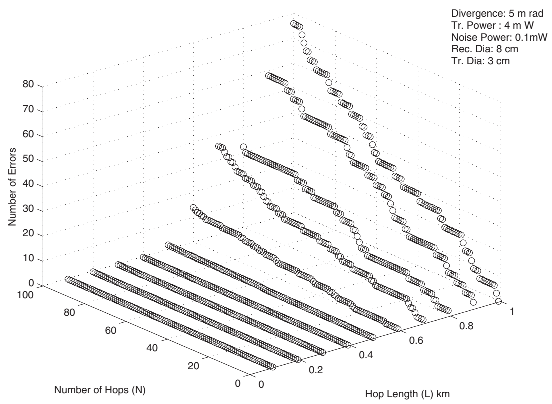

Fig. 7. Error accumulation with hop length.

representing clear weather to light rain conditions [6].The simulation results for both the single hop case and multiple hop case are presented. An end-to-end range of 2.5 KMs is choosen for the FSO link. In the case of a single hop, the distance between the transmitter and the receiver is 2.5 KMs. In the case of multiple hops, the range is divided into 5 hops, each hop being 500 meters. The end-to-end power used for both the cases is set to be equal. The end-to-end error distribution for a single hop case is shown in Figure 8(a) and for a multi-hop case is shown in Figure 8(b). The mean BER of the single hop is more than that of multi-hop. The error in the case of single hop is more widely distributed than in the case of a multi-hop. (For specific values, please refer to Table I.)

B. *Adverse Weather Conditions*

For adverse weather conditions, the visibility is taken as a Gaussian with a mean at 3 KMs and a variance of 1.5 Kms, representing moderate to heavy rain/snow and light fog conditions [6]. The end-to-end range is 2.5 Kms for a single hop and 5 hops with hoplength of 500 meters in the case of multi-hop scenario. The end-to-end error distribution for single hop scenario is illustrated in Figure 9(a). As seen the error is widely distributed causing the variance to be very high. Designing such an FSO link to operate reliably over wide range of visibilities is a challenge and also inefficient.

Figure 9(b) illustrates the end-to-end error distribution in the case of multi-hop operation. The error is contained within a small region, making the variance considerably small. The reliability of such an FSO link can be increased easily and efficiently compared to the single hop case.

Clearly, there is an improvement in both the mean and the variance in the case of multiple hops. A comparison of the mean error and the variance for both the single hop and multi-hop cases is given in the Table I.

V. MULTI-HOP SYSTEM: AMPLIFIED RELAYING

In amplified relaying, each intermediate terminal simply amplifies the received signal from the immediately preceeding terminal. Due to this, the noise also gets amplified by each intermediate terminal and hence is propagated end to end. Any error due to decoding thus is present only at the end receiver and the delay due to relaying by the intermediate terminals is minimized.

At each hop, the received signal plus noise is amplified. Hence, the received SNR is same as the transmitted SNR. Noise gets added to the signal at each hop by N_o. For N hops, the received signal can be expressed as:

$$SNR_{NthHop} = \frac{SNR}{N \cdot N_o}$$

The error behavior for such systems is work in progress; it can be shown that the BER gain and the variance gain in the case of the amplifying system is smaller than that for the decoding system.

VI. CONCLUSIONS AND FUTURE WORK

We demonstrated that error performance of the multi-hop free space optical communication is better than single hop communication for the same end-to-end link range and the same end-to-end power. We showed that the mean error rate in the case of multi-hop is smaller than that of the single hop equivalent, for both clear weather and adverse weather conditions.

More importantly, the variance of the error rate is significantly smaller for multi-hop operation. This narrow variance of the error helps to design effective FEC codes for the multi-hop network. This approach is more energy efficient to improve the FSO link reliability since the range of the target error rates is smaller as compared to single hop operation.

As the number of hops is increased, the error behavior improves, at a cost of increased delay at each hop. Optimizing the tradeoff between errors and end-to-end delay in a multi-hop scenario an interesting problem for future work. Similutaneously optimizing the system reliability, given constraints on the overall system infrastructure costs in multi-hop scenario is another interesting future problem.

VII. ACKNOWLEDGMENT

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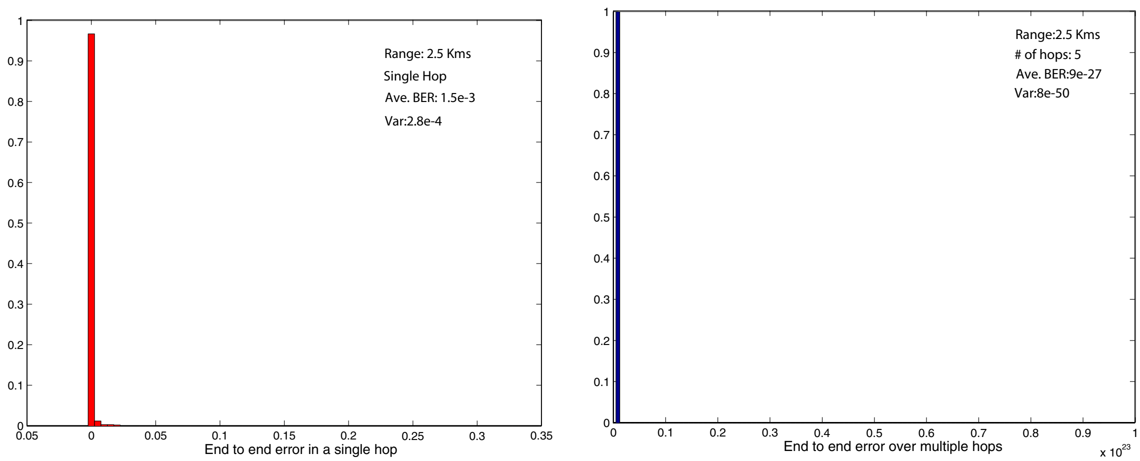
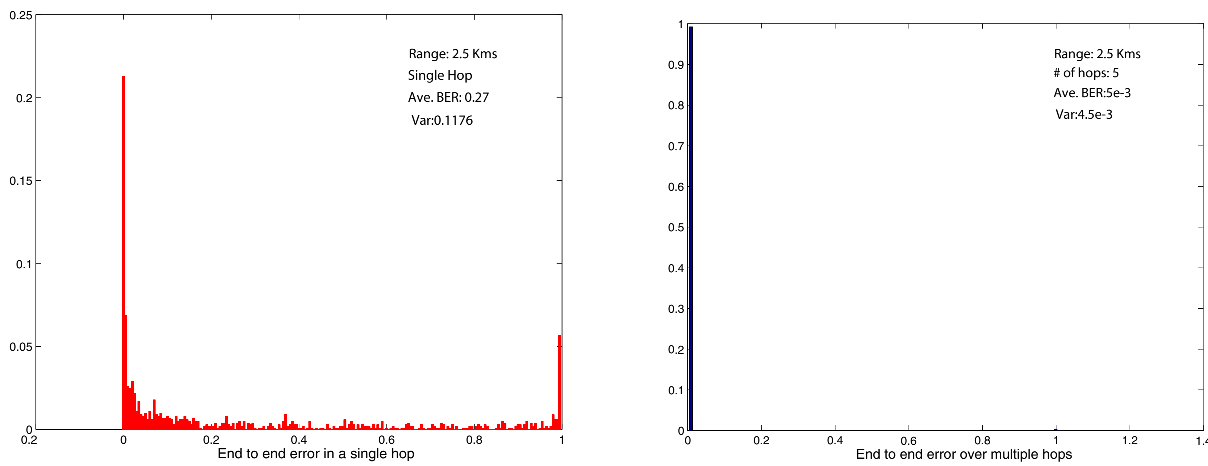


(a) (b)

Fig. 8. Error distribution for clear weather conditions: (a) Single hop FSO link (b) Multi-hop FSO link.



(a) (b)

Fig. 9. Error distribution for rainy/snowy weather conditions: (a) Single hop FSO link (b) Multi-hop FSO link.

Number of hops	Clear Weather Mean BER	Adverse Weather Mean BER	Clear Weather BER Variance	Adverse Weather BER Variance
1	1.5e-3	0.27	0.02	0.1176
5	9e-27	5e-3	8e-50	4.5e-3

TABLE I

COMPARISON OF MEAN BER AND BER VARIANCE FOR SINGLE HOP AND MULTI-HOP SCENARIOS.