Multi-channel Communication in Free-Space Optical Networks for the Last-mile

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Abstract— **Free-Space Optical communication technology is a potential solution to the last mile or broadband access problem. Conventional free-space optical (FSO) communication is over a single link between two nodes. We explore multi-channel FSO communication system using compact (a maximum of a Sq.Ft) 2-dimensional antennas with multiple communication links between them to achieve very high aggregate bandwidths (100's of Gbps). But, close packaging of optical channels on the arrays causes inter-channel interference, reducing per channel capacity. We model the error due to inter-channel interference for such arrays and estimate the channel capacity. We address the multi-channel interference issue by both array design and by employing optical orthogonal codes (OOCs) for free-space optical communications and show that we can achieve multi Gbps bandwidths using such arrays. Possible applications for such multi-channel FSO systems can be in multi-hop broadband access networks or mesh networks and in back haul, connecting wireless base stations.**

I. Introduction

Today, there is a tremendous need for a broadband wireless access technology that can support the high bandwidth requirements of a last-mile wireless broadband access network, for example, in a Wireless Mesh Networks (WMNs) or wireless backbone for metro/urban area networks. In areas without pre-existing physical cable or telephone networks, FSO and WLAN are the two viable alternatives for broadband access. Free-Space optical networks [1], [2], can effectively complement RF-based WLAN technologies like 802.11b/a, and WMAN technologies like 802.16 for the last mile or broadband access problem. A single FSO link can provide a bandwidth up to a few Giga bits per second.

Traditionally, free-space optical (FSO) communications use a single transmitting antenna (laser/VCSEL/LED) and a receiving antenna (a photo-detector) for single channel communication [2] between two nodes. In this paper we explore multi-channel free-space optical communication system using 2-dimensional multi-element optical antennas. Unlike in RF, by "channel", we mean spatial channel rather than frequency channel. Multi-element array design for FSO communication is very attractive since it offers high aggregate bandwidth and link robustness due to spatial diversity.

We are interested in compact (a maximum of a Sq.Ft) 2-dimensional array antennas capable of achieving very high aggregate bandwidths (a few Gbps) using FSO communications [3] as shown in Figure 1. Future applications for such

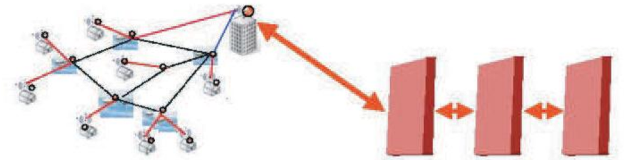

Fig. 1. An Example Illustrating the Application of 2-D FSO Arrays in Back-haul.

multi-channel FSO systems can be in multi-hop broadband access networks or mesh networks and in back haul, connecting wireless base stations. Ideally, the bandwidth offered by a multi-channel FSO system should increase with the number of channels. As an example, optical transceivers are capable of operating at bandwidths around 1 Gbps. With each transceiver operating at a speed of 1 Gbps, a 10×10 array will give 100 Gbps in aggregate capacity. But close packaging of transceivers on the arrays causes inter-channel interference, thereby reducing per channel capacity due to finite divergence of the light beam. We model the error due to inter-channel interference for such arrays and estimate the channel capacity. In the paper [3], we obtained the expression for the error probability specific to uniform array design. In this paper we obtain a general expression for the error probability due to inter-channel interference, independent of the array design. We address the multi-channel interference issue by both array design and by employing (OOCs) for free-space optical communications and show that we can achieve multi Gbps bandwidths using such arrays.

The rest of the paper is organized as follows: In Section II, we introduce the FSO communication system, and the channel model. We also obtain the expression for bit error probability for un-coded signals due to inter channel interference. In Section III, we obtain the expression for probability of bit error due to inter channel interference for orthogonally coded signals. In Section IV-A, we introduce the multi-element free-space optical array antennas we are considering in this paper and compare them in terms of the channel capacity and error probability. We also show how an improvement in capacity can

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be achieved using optical orthogonal codes(OOCs). Section VI concludes the paper.

II. System Description and Channel Model

In a typical single channel FSO communication system, the transmitter is a modulated light source, typically a low-powered laser operating in infrared band. The receiver is a photo-detector, and outputs a current proportional to the received light intensity. The receiver is in line of sight of the laser beam from the transmitter.

FSO communication supports duplex connection, therefore both transmitter and receiver are present at both the ends. We call each end an “optical transceiver”, which can both transmit and receive at the same time. The intensity of the light varies across the cross section of the light beam [2] following the Gaussian beam profile. Free-space optical communication uses On-Off Keying (OOK) for transmitting the information bits. On-Off keying is a digital modulation method, where in the amplitude of the carrier is switched on and off. The on condition corresponds to a code 1, and the off corresponds to a code 0. At the end of a bit period, the receiver’s output is compared to a set threshold value, say I_T and a decision on the transmitted symbol is made to be a code 0, if the receiver’s output is less than I_T or a code 1 if otherwise. If the receiver’s output is equal to the threshold value, then an arbitrary choice of 0 or 1 is made.

The signal from the transmitter can be expressed as:

$$S_{OOK} = \begin{cases} S_1(t) = 0 & (0 \leq t \leq T_b, binary0) \\ S_2(t) = Acos(\omega_0 t + \theta_0) & (0 \leq t \leq T_b, binary1) \end{cases}$$

In a single channel FSO communication system, the received signal quality is limited by Gaussian shot noise following the photo-detector [4]. In the presence of such a Gaussian noise with a power spectral density of $\mathcal{N}_0$ the signal to noise ration (SNR) is given by:

$$\varepsilon = (\frac{A^2 T_b}{2} \cdot \frac{1}{2} + 0 \cdot \frac{1}{2})\frac{1}{\mathcal{N}_0} = \frac{A^2 T_b}{4\mathcal{N}_0}$$

However, in a multi-channel system with $K+1$ simultaneously operating channels, like in a 2-dimensional array or a 3-dimensional sphere, the received signal is distorted by both the above described Gaussian noise and the inter-channel interference. In this section, we obtain the expression for the error caused by the combined affect of the Gaussian channel noise and inter-channel interference.

The received signal $r(t)$ can be represented as:

$$r(t) = s(t) + \eta + \zeta$$

where η is the Gaussian noise due to thermal noise and ζ is the inter-channel interference from K undesired users. This can be equivalently written as

$$r(t) = \sum_{k=0}^{K} s_k(t) + n_k(t)$$

For un-coded synchronized multi-channel use, each of the desired user’s bit is overlapped in time by K undesired users. The interference caused by each of these K users can be modeled as a bernoulli random variable. For a large K, we can approximate the distribution of the inter-channel interference as a Gaussian random variable invoking Central Limit Theorem (CLT).

Let us combine η and ζ into a single Gaussian random variable ξ. I.e.,

$$\xi = \eta + \zeta$$

Then the error probability for free-space optical communications with on-off keying is defined as:

$$p_e = p(\xi \geq I_T) \cdot p(\text{The desired user transmits a } 0)$$

This is because, an error occurs only when the signals from the undesired users contribute a code 1 AND, when the signal from the desired user is code 0, since optical pulses are either positive or zero and at the receiver and we use an energy threshold detector. Therefore, we model the array communication channel as a binary asymmetric channel (BAC) and estimate its capacity as a function of bit error probability. We study the behavior of the channel capacity with package density on each of the arrays, distance between arrays and divergence angle of the light source used for communication.The capacity of such a channel is known to be:

$$C = max_{p_1} H(\bar{p_1}\bar{P_e}) - p_1 H(\bar{P_e})$$

where C is the channel capacity, H denotes entropy, p_1 is the input symbol (*ONE* or *ZERO*) probability distribution, and P_e is the probability of error.

We derive the expression for P_e for the array communication system below. We fix the input symbol distribution p_1 at 0.5, and estimate the channel capacity as a function of P_e, which in turn depends upon array design parameters such as transceiver package density, light source divergence, and distance between the arrays.

For an OOK transmitter with equal symbol probabilities, this is:

$$p_e = \frac{1}{2}\int_{I_T}^{\infty} p_\xi(x)dx$$

$$= \frac{1}{2}\int_{I_T}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_\xi} e^{-(\frac{x^2}{2\sigma_\xi^2})} dx$$

$$= Q(\frac{I_T}{2\sigma_\xi})$$

σ_ξ^2 is the variance of the sum of noise and interference. Therefore,

$$\sigma_\xi^2 = \sigma_\eta^2 + \sigma_\zeta^2$$

Now, we need the variance of the interference to compute the bit error. The variance of the interference from the K undesired users is:

$$var(\zeta) = E[(\sum_{k=0}^{K-1} I_k)^2]$$

where I_k is the intensity received from k^{th} interferer given by:

$$I_k = I_0 e^{-(\frac{4\theta_k}{\theta})^2}$$

Where, θ_k is the angle of transmission from the kth interferer and θ the divergence angle of the laser beam.

Therefore,

$$var(\eta) = E[(\sum_{k=0}^{K-1} I_0 e^{-(\frac{4\theta_k}{\theta})^2})^2]$$

$$= \sum_{k=0}^{K-1} E[I_0^2] e^{-(\frac{4\theta_k}{\theta})^2}$$

Since I_0 is Bernoulli distributed with $p = 0.5$,

$$E[I_0^2] = 0.5$$

Substituting,

$$var(\eta) = \frac{1}{2} \sum_{k=0}^{K-1} e^{-(\frac{4\theta_k}{\theta})^2}$$

And

$$\sigma_\xi^2 = \frac{1}{2} \sum_{k=0}^{K-1} e^{-(\frac{4\theta_k}{\theta})^2} + \frac{N_0 T_b}{4}$$

Therefore, the probability of error for multi-element free-space optical communications is given by:

$$p_e = Q(\frac{I_T}{2\sqrt{\frac{1}{2}\sum_{k=0}^{K-1} e^{-(\frac{4\theta_k}{\theta})^2} + \frac{N_0 T_b}{4}}}) \quad (1)$$

Thus, we obtain the bit error probability due to inter-channel interference for communication between multiple element antennas in a free-space optical communication system. In the next section, we examine how we can improve this error performance using optical orthogonal codes.

III. Optical Orthogonal Codes

An optical orthogonal code (OOC) is a family of **(0,1)** sequences with good auto- and cross-correlation properties, i.e., the autocorrelation of each sequence exhibits the "thumbtack" shape and the cross correlation between any two sequences remains low throughout. Its study has been motivated by an application in a code-division multiple-access fiber optical channel [5], [6]. The use of OOCs enables a large number of asynchronous users to transmit information efficiently and reliably. The thumbtack shape of the autocorrelation facilitates the detection of the desired signal, and the low cross correlation reduces the interference from unwanted signals in the network. We apply theses codes for free-space optical communications for the first time and study their performance in the use of multiple element antennas to reduce inter-channel interference, there by increasing the aggregate bandwidth provided by these antennas.

When using OOCs, there are $K+1$ transmitter and receiver pairs. The OOCs essentially become a set of address codes between each of these pairs. To send information from user

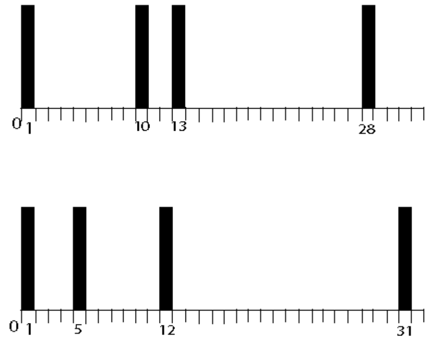


Fig. 2. Two Optical Orthogonal Codes with weight N =4, length F = 32 and $\lambda_a = \lambda_b = 1$.

j to user k, the address code (OOC) is impressed upon the data by the encoder at the jth element of the optical antenna. At the receiver, the desired optical signal is recovered in the presence of all other users' optical signals.

Let $x(t)$ and $y(t)$ be two periodic signals which can be expressed as [5], [6]

$$x(t) = \frac{1}{T_c} \sum_{n=-\infty}^{\infty} x_n P_{T_c}(t - nT_c)$$

$$y(t) = \frac{1}{T_c} \sum_{n=-\infty}^{\infty} y_n P_{T_c}(t - nT_c)$$

where P_{T_c} is a unit rectangular pulse of duration T_c. For $x(t) = x(t+T)$ and $y(t) = y(t+T)$ for all t, then the sequences (x_n) and (y_n) are periodic sequences with period $F = \frac{T}{T_c}$.

The code sequences are further defined by N, which is the weight of the OOC sequences, the auto-correlation constant λ_a, and the cross-correlation constant λ_c as illustrated in Figure 2. Strict orthogonality would require that $\lambda_a = \lambda_c = 0$. Since optical signals form a positive system $(0, 1)$, such signals cannot be optically manipulated to add to zero with other signals, if coherent interference effects are eliminated. Here, we consider those families of OOCs for which their auto- and cross correlation constraints $\lambda_a = \lambda_c = 1$.

Now, we will obtain the expression for the error probability in the case of OOCs for the FSO channel with Gaussian noise. In the previous section, the general expression for the error probability due to interference in the presence of Gaussian noise is given as:

$$p_e = Q(\frac{I_T}{2\sigma_\xi})$$

where

$$\xi = \eta + \zeta_{ooc}$$

and η is the Gaussian noise due to thermal noise and ζ_{ooc} is the inter-channel interference from K undesired users when orthogonal codes are used. When orthogonal codes of weight N and length F are used for communication, the variance of the interfering signal

$$var(\zeta_{ooc}) = E[(\sum_{k=0}^{K-1} I_k)^2]$$

where

$$I_k = I_0 e^{-(\frac{2y_k}{(W_d)_k})^2}$$

and for orthogonally coded signals, I_k is given by:

$$I_k = b_n(t) DP_k(t)$$

and

$$b_k(t) = \sum_{l=-\infty}^{\infty} b_l^n P_T(t - lT)$$

where $b^n = b_l^n$ is the n^{th} data sequence that takes a 0 or 1 (on-off keying) for each l with equal probability. And DP_n is the n^{th} user's OOC sequence.

The variance of such a signal after applying CLT for Gaussian approximation is given by:

$$var(\zeta_{ooc}) = \sum_{k=0}^{K-1} E[I_0^2] e^{-(\frac{4\theta_k}{\theta})^2}$$

$$= (\frac{N^2}{2F})(1 - \frac{N^2}{2F}) \sum_{k=0}^{K-1} e^{-(\frac{4\theta_k}{\theta})^2}$$

The probability of error after using optical orthogonal codes is then given by:

$$p_e = Q(\frac{I_T}{2\sqrt{(\frac{N^2}{2F})(1 - \frac{N^2}{2F}) \sum_{k=0}^{K-1} e^{-(\frac{4\theta_k}{\theta})^2} + \frac{N_0 T_b}{4}}}) \quad (2)$$

In Equation 2, when the threshold I_T is greater than the number of undesired signals $K+1$, the error ideally becomes zero. Since we have used Gaussian approximation, the error becomes very small instead of becoming zero for smaller number of undesired signals ($\sim$10).

IV. Multi-Element Optical Antennas

We study FSO communication between 2-dimensional arrays with multiple transmitters and receivers and how the capacity of such a channel varies with the system parameters like the distance between the arrays, divergence angle of the light source and package density of the transceivers on the surface of the array. We show that by carefully choosing the pattern with which the transceivers are packed on the array, the capacity of the channel can be improved. In addition, by implementing optical orthogonal codes on these arrays, capacity close to 1 can be achieved when the codes are chosen properly.

We choose two array designs, one in which the transceivers are uniformly distributed spatially on the array. The second one, where in the transceivers are distributed following a helical arrangement. We show that this non-uniform distribution gives better capacity with increasing link range or source divergence.

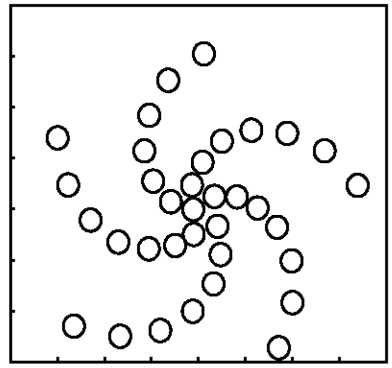


Fig. 3. Helical Array Design.

In the next two sections we describe these optical antennas more in detail and study how the error probability and in turn the capacity behaves for these two structures. For the two designs of optical antennas, the basic question is to find "K", the number of interfering users/channels causing inter-channel interference as a function of the design parameters. Once we find that, Equation 1 gives the error for each of these designs when the orthogonal codes are not used. For this, we can obtain the capacity for the multi-element optical antennas. And then, using Equation 2, we obtain the capacity of the channel when orthogonal codes are employed.

The 2-dimensional arrays we propose for FSO communications are shown in Figure 3 and Figure 4. The circles denote the optical transceivers, i.e. a light source (Laser/LED) and a photo-detector. Multiple such transceivers are spaced on the array. The total number of transceivers per unit area on an array is referred to as *package density* ρ.

Two such identical arrays face each other to facilitate communication between the corresponding optical transceivers on the arrays. Because of the finite transceiver angle, the light signals transmitted will diverge by the time they reach the opposite array and they are not only received by the corresponding transceiver on the opposite array, but also by its neighboring transceivers, causing inter-channel interference or cross talk.

For example, as shown in Figure 4, consider the transmission from the transceiver T_0 on the array A (T_0^A), to T_0 on the array B (T_0^B). Because of the finite transceiver angle θ, a cone from the transceiver T_0^A extends onto the array B and defines the field of view of the transceiver. The radius of the cone on the array B is a function of the distance between the two arrays d and the transceiver angle θ as given by:

$$r = dtan(\theta)$$

Due to this, not only T_0^B is present in T_0^A's field of view, but also four more transceivers T_1^B, T_2^B, T_4^B, and T_7^B causing interference at those other transceivers.

A. Array Designs: Helical Vs Uniform Distribution of Transceivers

In this section we study the two array designs we considered for this paper and compare them in terms of error probability

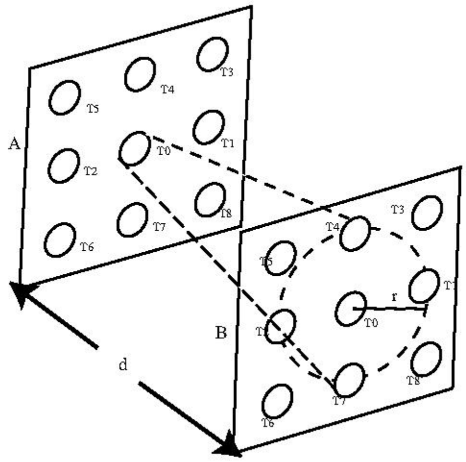


Fig. 4. Two Communicating Arrays.

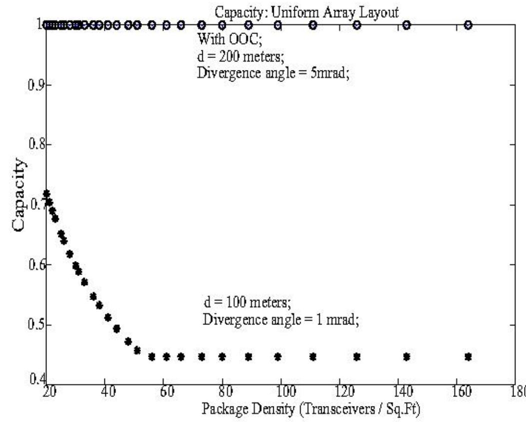


Fig. 5. Capacity of Uniform Arrays.

due to interference, equivalently, the channel capacity. We simulated the arrays in Matlab, with package density varied from 0 transceivers per Sq.Ft to $\sim$ 120 transceivers per Sq.Ft. We computed the number of interfering channels for a given divergence angle θ of the light source used and inter-array distance d, and estimated the error probability P_e. We then estimated the channel capacity for the BAC using P_e. We assumed equal transmission probability for a *ONE* and *ZERO* ($p_0 = 1/2$).

We made the following observations from our simulations. Helical arrays have a relatively lower error probability compared to uniform arrays for a given distance, divergence angle, and the package density. And the reason for that is, for a given package density on the array, the number of interferers is higher for the uniform array compared to the helical arrays due to the non-uniform placement of the transceivers on the array. Uniform arrays without OOCs, may be used with narrower light sources (1 mrad) like lasers and only over shorter distances (50 -75 meters), since the error due to inter channel interference is high even at shorter distances. On the other hand, with helical arrays we can achieve lower bit error rates at higher divergence angles and over longer link ranges, making the helical arrays more practical to use. When we implemented OOCs on the arrays, the arrays can be used over 500 meters and with inexpensive components like semiconductor lasers e.g. VCSELS, this is illustrated by the improved error probability in Figure 6(a). In general, as the package density increases, the error probability increases and hence the capacity decreases. The specific package density at which the capacity drops from 1 is a function of the distance between the arrays, and the angle of the transceivers and the specific arrangement of the transceivers on the array. When we achieve near ideal capacity, for a package density of 100 transceivers per Sq. Ft and transmitters operating at data rates of 500Mbps to 4Gbps, we can realize bandwidths of $50 - 400$Gbps using the 2-dimensional arrays.

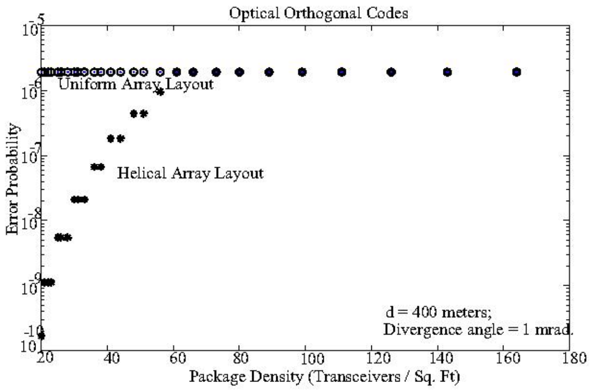
Figure 5 shows the capacity that can be achieved using uniform distribution of transceivers on the arrays. Uniform arrays have high inter channel interference at relatively lower distances and divergence angles. This can be improved with the use of orthogonal codes.

In Figure 6 per-channel capacity with package density for helical arrays is illustrated. As the package density increases, the error probability increases and hence the capacity decreases. When OOCs are implemented, we achieve near ideal channel capacities , and hence very high aggregate bandwidths. An array without OOCs, can be used only with lasers and for shorter distances ($\sim$ 150 meters); where as when we implement OOCs on the arrays, the arrays can be used over $\sim$ 500 meters and with VCSELS.

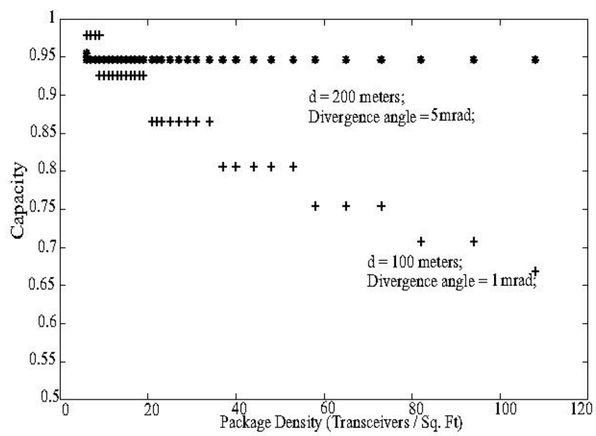
V. Bandwidth-Volume Product (BVP) and Array Design Guidelines

We define the performance of an FSO communication channel by three design parameters: (i) number of channels per array, (ii) the capacity of each of the channel in bits per second, and (iii) the distance over which the arrays can communicate with that capacity. We define a useful design metric that incorporates all the above parameters of the system as a product. We designate it as *Bandwidth-Volume Product (BVP)*. The advantage of BVP is that it provides an integrated performance evaluation measure to aid the design of the arrays, when choosing various parameters (e.g. d, θ) of the multi-element FSO system. Here, "Bandwidth" denotes the capacity of a single channel, i.e. the unit of Bandwidth is *Mbps*. By 'volume' we mean the volume of space between the two planar arrays which is defined by the number of channels on the array and the communication distance, therefore, the unit of the Volume here is *meter*. This means that the unit of BVP is Mbps-meter.

Bandwidth-Volume Product gives the "number of useful bits" over the range specified. BVP is synonymous to the "Bandwidth-Distance Product" metric of a fiber-optic link. In the case of a fiber-optic link, it is the fiber dispersion that adversely effects the aggregate capacity, whereas in the multi-channel FSO link, it is the interference.



(a) Improvement of error probability with OOCs.



(b) Capacity for the helical array.

Fig. 6. Array Capacity

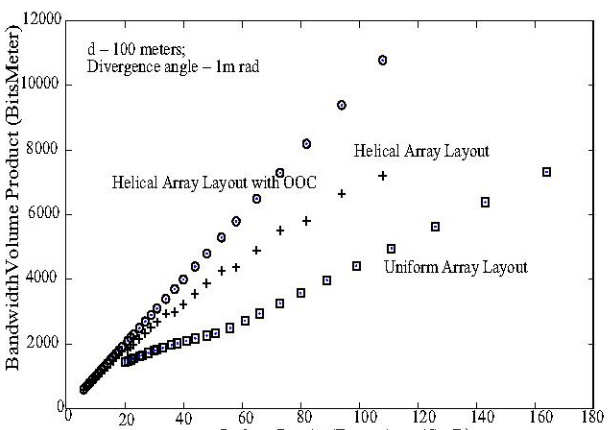


Fig. 7. Bandwidth-Volume Product.

Figure 7 shows the Bandwidth-Volume Product (BVP) for the arrays. The BVP plot provides the design choices for a given array design or for a desired package density. As the package density increases, BVP for various arrays first increases and then decreases. The point at which BVP decreases, the per channel capacity of the arrays drops drastically due to inter-channel interference. In the case of helical arrays, the BVP drops much more slowly. A comparison of BVPs for uniform arrays, helical arrays, and helical arrays with orthogonal coding is shown. As seen, helical arrays with orthogonal coding have near ideal performance, it does not drop for even the package densities as high as 100 transceiver per Sq.Ft and over long distances ($\sim$ 400 meters).

From the above it is clear that non-uniform placement of transceivers on the array, for example, a helical array performs better than uniform distribution of transceivers. Helical arrays achieve higher per channel capacity, and hence higher aggregate bandwidths for a given package density and communication range between transceivers. The additional cost of implementing the OOCs is paid off in terms of increased channel capacity and longer operating ranges.

VI. Conclusions

We demonstrated that multi-channel systems for free-space optical (FSO) communications give excellent bandwidth performance providing over a few 100 Gbps. In this paper, we considered two designs for the 2-dimensional arrays for analysis. An interesting future problem is to find an optimal design for the array that achieves highest capacity for a given range, transmitter divergence, and the number of transmitters. Multiple hops using such FSO antennas can be easily implemented in a LAN environment. For example, in an indoor access network or a campus-wide LAN scenario or in a mesh network, we can tremendously increase the bandwidth by using 2-dimensional arrays. To use these arrays over very long distances outdoors, we would need very narrow beams coupled with auto-aligning mechanisms.

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