Modeling for Pricing of Loss-rate Guaranteed Internet Service Contracts

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Abstract—The technological advancements in recent years are allowing Internet Service Providers (ISPs) to provide quality of service (QoS) assurances for data flow through their domains. Depending on the application, QoS may be necessary not only as a bandwidth assurance, but also on the loss, delay, etc., experienced by the data. In the Internet technologies, such guarantees may only be provided in probabilistic terms. In this article, we develop a stochastic modeling framework for the pricing of probabilistic loss-rate guaranteed Internet service contracts.

I. INTRODUCTION

The Internet today mostly provides a best-effort service, i.e., it tries its best to push the data through from a source to a destination. However, in doing so it does not give any guarantees to its customers regarding the data actually reaching its destination. Significant improvements in the network technology over the past few years are enabling the Internet Service Providers (ISPs) to incorporate better assurances on Quality of Service (QoS) for the traffic within their network domains. When the traffic crosses the domain's boundaries, it is back with a best-effort service, with no assurances. But mechanisms can be developed so that the providers leverage on their network resources and improve utilization by pricing bandwidth appropriately and providing customers with assured services for their inter-domain traffic.

In this article, we develop models for a spot-pricing framework for *intra-domain* assured bandwidth services, specifically for expected bandwidth with a loss-rate guarantee. The framework lays the foundation for pricing *inter-domain* guaranteed bandwidth for enterprise customers. A dynamic pricing scheme is employed, in which prices are generated responding to customer demand characteristics and the current state of the network. An attractive feature of the framework is that it is implementable on the differentiated services architecture (diffserv) and can be overlayed on schemes which are capable of providing intra-domain assured services, such as, Distributed Dynamic Capacity Contracting [1].

The article is organized as follows. Section II provides a brief review of state-of-the-art for bandwidth pricing, advancements for supporting QoS towards the realization of assured bandwidth provision. In section III, we describe the models for spot-pricing in detail. Section IV presents some simulation modeling results. In section V, we will summarize our discussions, pointing the steps for our future research.

II. LITERATURE REVIEW AND BACKGROUND

A. Bandwidth Pricing

Internet pricing is a growing area of research. Until recently, the providers had, in general, opted for a flat rate or time-of-the-day pricing [2][3]. These schemes do not react to the current state of the network. If transmission of data through the network causes congestion, this is not reflected in the prices. These static pricing schemes can work satisfactorily in the current market, because of slower access bandwidth to customers, improvements coming from deployment of faster routers, better routing algorithms, and upgrades from copper to optical links. In the long-run, at least the specialized Internet services will need dynamic pricing.

On the other hand, dynamic pricing schemes such as Smart Market [4], Proportional Fair Pricing Scheme [5], Priority Pricing [6] take the state of the network into account. However, practicality of the implementation of these pricing strategies is in question due to their fine time-granularity. Recently, an implementable Pricing Over Congestion Control (POCC) [7] for diff-serv architecture has been proposed which can be overlaid on the congestion control framework proposed by Harrison, et al. [8] and provides a range of fairness in rate allocation by using pricing as a tool.

Pricing approaches for telecom bandwidth contracts has also received attention in the past years. These contracts are longer term contracts than those discussed in the Internet pricing literature. For end-to-end bandwidth pricing, the role of geographical arbitrage and application of compound option techniques are investigated [9] [10] [11] [12] [13] [14] [15]. However, guarantees on stochastic QoS measures, as observed in the Internet technologies, are considered only to a limited extent.

B. Technology to Support Quality of Service

In contrast with a leased line or a circuit-switching setting, in packet-switching traffic is not perfectly isolated due to the nature of scheduling mechanisms employed in the Internet [16] [17] [18] [19]. Close monitoring and traffic engineering mechanisms are set in place to effect the QoS delivery.

QoS deployment in multi-domain, IP-based inter-networks has been an elusive goal partly due to complex deployment issues [20]. Therefore, from an architectural standpoint, contemporary QoS research has recognized the need to *simplify* and de-couple building blocks to promote implementation and inter-network deployment. The int-serv and RTP work [21]

[22] de-coupled end-to-end support from network support for QoS. RSVP de-coupled inter-network signaling from routing. MPLS [23] de-coupled forwarding mechanisms from the routing control plane, leading to traffic engineering capabilities [24]. The diff-serv services [25] [26] [27] and core-stateless fair queuing (CSFQ) [28] further simplified core architecture and moved data-plane complexity to the "edges," and allowed a range of control-plane options[23] [24] [29] [30]. Therefore, concepts are being developed to address the challenge of provisioning QoS assurances at various levels - management of packets, configuration of inter-networks, and service delivery modes to customers [31] [32] [33] [34]. Pilot studies are in progress that test these concepts [35].

III. SPOT PRICING FRAMEWORK

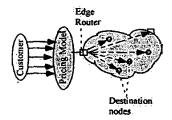


Fig. 1: Basic pricing model implemented at an access point

Network performance can be defined in terms of a combination of its bandwidth, delay, delay-jitter, loss properties. Based on these performance measures, QoS guarantees can be stated in deterministic or probabilistic terms. In this article, we will focus on an expected level of bandwidth with loss-rate guarantees.

Provision of QoS guaranteed contract is made at an access (edge) or exchange point. Such models implemented at the access and/or exchange points of different domains will allow the creation of inter-domain service assurance to the customers. The basic intra-domain bandwidth pricing model is shown in Figure 1.

At the core of our spot-pricing framework is a nonlinear pricing scheme. The term *nonlinear pricing* refers to a pricing scheme where the tariff is not proportional to the quantity purchased and the marginal prices for successive purchases decrease [36]. The advantages of nonlinear pricing stem from the heterogeneity among the populations of customers.

Unlike a linear or uniform pricing scheme, where the price schedule or the marginal price is constant irrespective of the customer's demand, under a nonlinear pricing scheme, prices are chosen according to the inverse of the price elasticities for each incremental quantity purchased, and therefore, are decreasing along the customer's demand. Prices are also set above the marginal costs in order to recover the provider's full operating and capital expenses. Considerations of cost, competitive pressures and profits constitute the major motivations for favoring a nonlinear pricing scheme. Nonlinear pricing is particularly relevant in industries where large fixed cost is involved, as by favorable pricing a provider can attract

customers with large demand and thus improve utilization of its capacity and sufficiently recover the fixed cost. A well known example of nonlinear pricing, Ramsey pricing, is briefly described next; more details may found in Gupta et al [37].

A. Ramsey Pricing Model

Ramsey pricing has been widely popular in the telecommunication and power sectors [38][36]. It produces an efficient tariff design in situations where due to either regulation or competition, revenues sufficient to only recover the provider's total costs are achievable. The guiding principle of the Ramsey pricing model is to develop tariffs that maximize an aggregate of customer's benefits, subject to the constraint that the provider's revenues recover its total costs (fixed as well as variable). An additional constraint of the model is that the price schedule calculated from it must not exceed a uniform price schedule which provides the same net revenue to the provider. This second constraint addresses the welfare issue. It ensures no customer is worse off with Ramsey pricing, which in practice establishes Ramsey pricing as an improvement for both the provider and the customers.

Demand characteristics of a population are usually described by a demand profile, N(p(q),q), defined as the number or fraction of customers who will buy at least q units at the marginal price p(q). The fraction of population interpretation of the demand profile is followed in this article. The price schedule obtained from Ramsey pricing maximizes a commonly used measure for the aggregated customers' benefits, the total consumer surplus, given by $\mathrm{CS}(q) = \int\limits_{p(q)}^{\infty} N(p,q) dp$. Therefore, if p(q) is the optimal price schedule and c(q) is the

Therefore, if p(q) is the optimal price schedule and c(q) is the marginal cost for the q^{th} unit, and $\eta(p(q), q)$ is the elasticity of the demand profile, the optimal price schedule should satisfy the Ramsey rule:

$$\frac{p(q)-c(q)}{p(q)} = \frac{\alpha}{\eta(p(q),q)} \tag{1}$$

where the Ramsey number α is the fraction of the monopoly profit margin common to all units of customers' purchases that is needed by the provider for purpose of cost recovery. Ramsey number indicates the nature of firm the provider is. A larger value of α relates to a higher revenue requirement or monopoly power. For instance, a profit-maximizing monopolist has $\alpha=1$, while a regulated firm with no binding revenue requirement has an $\alpha=0$. In case of a budget-constrained welfare maximization and an oligopolistic competition, $0<\alpha<1$.

B. Model Definition and Assumptions

In this section we describe our spot pricing model for the intra-domain assured bandwidth contracts. Customers purchase bandwidth contracts of a fixed duration T, for simple and immediate file transfer applications. Literature on data analysis of Internet traffic describes arrival of different flows to follow a Poisson process and file-sizes are best represented by heavy-tailed distributions [39] [40][41]. We model customers' arrival by a Poisson process at a rate of $\lambda = 5/min$ averaged over a

day. Arrivals are time dependent; based on historical data [42], we assume that 70% of the customers arrive between 7 a.m. and 5 a.m., 20% arrive between 5 p.m. and 11 p.m., while the rest 10% arrive between 11 p.m. and 7 a.m.

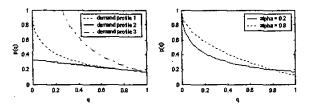


Fig. 2: Price schedule p(q) for demand profiles: (a) p(q) for different demand profiles ($\alpha=0.2$); (b) p(q) for different values of α

Upon arrival each customer announces its volume and loss-rate requirements to the provider. Based on the volume requested, the provider has to infer the load from the customer's traffic on the network. Using more information from the customer, such as number of users, type of applications, the provider assigns models for the customer's data arrival curve (Figure 3(a)). Specifically, the files arrival rate and file-size distributions are determined. File arrivals and sizes are modeled by Poisson and Pareto distributions, respectively, following the Internet traffic data analysis literature [43][40][44]. The file-size Pareto distribution has the probability density function, $P(x) = \frac{ab^a}{x^a+1}$, where the parameters are a = 0.35, b = 100. For simplicity, the parameters for the file-size distribution are kept fixed across customers.

The Asked Capacity (in Kbps) for the customer is then obtained by dividing the expected total volume requested by the contract duration, T. If the Asked Capacity is lower than the Available Capacity of the network at the time of arrival, the customer is accepted and a contract for bandwidth with required service levels is created. The customer is assigned the Asked Capacity and after time T the customer releases the capacity and leaves the system. If there is not enough capacity to accommodate the customer's demand, the customer leaves without being served. The Available Capacity is updated with every relevant event. The total Available Bandwidth for an access point is obtained by creating a single link abstraction for the network.

Modeling for the loss-rate characteristics for the customer's data is accomplished by identifying transfer time distributions for the customer's files. The distribution for file transfer times are also known to be heavy-tailed. The transfer times are modeled using Pareto distribution, where the parameters of the distribution depend on the size of the file being transferred.

Combining the file arrival rates, file sizes and transfer times, a service curve for the customer can be obtained. It should be noted, however, that this service curve is only a pseudo-service curve, since not all the data sent into the network by the customer is guaranteed to reach its destination. At any time, the amount of customer's data that is in the network is susceptible to losses.

The difference between the arrival curve and the pseudo-

service curve provides the amount of customer's data in the network at any time, called *data in-transit* (Figure 3(b)). The 95-th percentile of the data in-transit process is at 32Kb, where about 125 spikes are above this level. Data in-transit along with the state of the network are indicators of data-loss. The state of the network is depicted by an aggregate traffic process in the single link abstraction for the network. The aggregate traffic process is taken to have a cyclical pattern on a day time-scale along with noise that possesses self-similar characteristics [44] [45]. The loss-process is modeled as a two-state Markov process, where the transition probabilities depend on the current state of the network and the amount of customer's data in-transit.

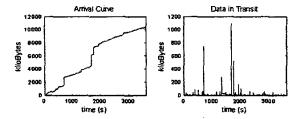


Fig. 3: Customer data flow over contract term: (a) Arrival curve; (b) Data in-transit

For each accepted customer, a price schedule p(q) is generated using the Ramsey pricing model and a given demanded capacity q. Demanded capacity is defined as the ratio of the Asked Capacity to the current Available Capacity, in order to scale q in the range [0,1]. A large Available Capacity relates to a smaller q value, which for a typical nonlinear pricing model indicates a higher marginal price p(q). The total price P(q) is computed by integration of the marginal price.

The provider is assumed to know the demand characteristics of the customers, for instance the arrival rate, contract durations, and the parameters for the Pareto distributions. As mentioned before, the total *Available Bandwidth* for an access point is obtained by creating a single link abstraction for the network and is taken to be fixed at 48 Mbps. This is only a representative choice, other time-variant choices can be accommodated in the framework.

C. Sample Demand Profiles

We consider 3 sample demand profiles [36] [37] for simulation implementation and analysis; each describes a different population of customers. A constant marginal cost, c(q) = c, taken as 0.2. Demand profile 1 represents a market where customers moderately react to price changes, $N(p,q) = 1 - \frac{q}{1-p}$, with the corresponding price schedule given as,

$$p(q) = 1 + \frac{q(1-\alpha)}{2\alpha} - \sqrt{(\frac{q(1-\alpha)}{2\alpha})^2 + \frac{q(1-c)}{\alpha}}.$$
 (2)

Demand profile 1 corresponds to a simple linear demand function $D(p) = \frac{1}{2} - \frac{1}{2}p$. The optimal uniform price for a profit-maximizing monopoly for this demand profile is $\frac{1+e}{2}$.

We will use this fact in simulation in order to compare a nonlinear pricing approach with a uniform pricing alternative.

Demand profile 2 has a linear relationship in terms of p and q, and relates to a quadratic demand function $D(p) = \frac{(1-p)^2}{2}$. It describes a market where customers are highly sensitive to price changes. Lastly, to analyze an extreme case of the market, we chose the third demand profile to describe a population that is extremely insensitive to price changes and is willing to pay a large range of quoted prices (See Figure 2).

It can also be shown that demand profile 1 dominates demand profile 2 for all possible values of p,q. Price schedules p(q) generated from demand profile 1 are higher than those for demand profile 2 for smaller values of q and become indistinguishable as q increases (Figure 2 (a)). It is true for all three demand profiles that a higher α value relates to a steeper variation of the price schedule p(q) with respect to q, and results in higher price schedule except for values of q close to 1 (Figure 2(b)).

IV. SIMULATION ANALYSIS AND RESULTS

We simulated our spot pricing model under different settings. Some of the simulation analysis and results are presented here. We implemented the 3 demand profiles described in the previous section in our spot pricing framework. Different types of providers are simulated by selecting different values for the Ramsey number, α . We simulated our pricing model in different network traffic loads described by a different combination of the average number of files, N, a customer sends and their contract duration, T. Four scenarios of the N,T combination were selected. Scenario 1 has N=1000,T=1

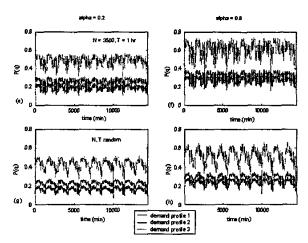


Fig. 4: Variation of total prices P(q): (a),(b) N=3500, T=1hour; (g),(h) $N \sim \text{Uniform}(1000,3500), T \sim \text{Uniform}(1,4)$ hours

hour and represents a network with low traffic load; Scenario 2 has N=3500, T=4 hours and corresponds to a network with high traffic load; Scenario 3 represents an extremely high instantaneous traffic load, with N=3500, T=1 hour; Scenario 4 has N uniformly distributed between 1000 and

3500, and T uniformly distributed between 1 and 4 hours, which is assumed to more realistically represent network traffic (Figure 4).

To compare a nonlinear pricing based framework with a linear pricing scheme, we simulated linear pricing specifically for demand profile 1 in Scenario 4 with N, T following the uniform distribution. Implementing the uniform pricing scheme implies that customers do not get served not only when there is not sufficient Available Bandwidth at the time of their arrival, but they also "balk" when the quoted price is higher than what they are willing to pay. This is because the uniform price is determined with the objective of meeting the provider's costs, therefore may not incorporate all customers' willingness to pay.

The primary measures for comparison of the results of simulation are the total price P(q) the provider charges each customer and the provider's revenues. An additional measure is the number of customers rejected or balked. The summary

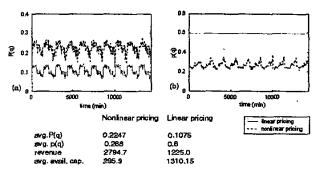


Fig. 5: Comparison of nonlinear pricing with linear pricing: (a) Comparison of total prices P(q); (b) Comparison of marginal prices p(q) with linear and nonlinear pricing

of the results are as follows, (1) the variation of prices in all simulated scenarios show a cyclical pattern with long-range dependence, further confirmed by an autocorrelation plot, (2) total prices for demand profile 1 and 2 follow similar patterns with the former uniformly dominating the latter, (3) while the 3^{rd} demand profile produces more variable and considerably higher prices. (4) The Ramsey number, α , affects both the variability and the levels of the total prices, however, does not significantly change the pattern in prices, (5) but the traffic load affects the pattern, the levels and variability of prices. And finally, (6) the average utilization and revenues are significantly higher for nonlinear pricing approach when compared with the uniform pricing scheme (Figure 5).

V. CONCLUSION AND EXTENSIONS

In this article we developed models for spot pricing for expected bandwidth contracts with loss-rate guarantees. By concatenating such fixed term contracts delivered at different access or exchange, inter-domain assured bandwidth contracts can be created. A nonlinear pricing model forms the core for pricing the expected bandwidth component of the contracts. In

our next steps, option pricing techniques will be employed for pricing of loss-rate and other delay, delay-jitter guarantees.

REFERENCES

- [1] M. Yuksel and S. Kalyanaraman, "Distributed dynamic capacity contracting: A congestion pricing framework for Diff-Serv," Proceedings of IFIP/IEEE International Conference on Management of Multimedia Networks and Services (MMNS). [Online]. Available: http://www.ecse. rpi.edu/Homepages/shivkuma/research/papers-rpi.html
- [2] A. Odlyzko, "Internet pricing and history of communications, AT&T Labs," 2000.
- "The economics of the internet: Utility, utilization, pricing and quality of service, AT&T Labs," 1998.
- [4] J. MacKie-Mason and H. Varian, "Pricing the congestible network resources," IEEE Journal on Selected Areas of Communications, vol. 13, pp. 1141-1149, 1995.
- [5] F. Kelly, A. Maulloo, and D. Tan, "Rate control in communication networks: Shadow prices, proportional fairness and stability," Journal of the Operational Research Society, vol. 49, pp. 237-252, 1998.
- A. Gupta, D. Stahl, and A. Whinston, Internet Economics. Boston, MA: MIT Press, 1997, ch. Priority pricing of Integrated Services networks, p. 323–352.
- [7] M. Yuksel, S. Kalyanaraman, and A. Goel, "Congestion pricing overlaid
- on edge-to-edge congestion control," GlobeCom (to appear), 2002.
 [8] D. Harrison, S. Kalyanaraman, and S. Ramakrishnan, "Overlay bandwidth services: Basic framework and edge-to-edge closed-loop building block," Poster in SIGCOMM, 2001.
- G. Cheliotis, "Structure and dynamics of bandwidth markets," Ph.D. dissertation, N.T.U. Athens, 2001.
- [10] S. Chiu, J. Crametz, and B. Null, The Telecoms Trading Revolution. London, UK: Risk Waters Group, 2000, ch. Commodity pricing fundamentals and telecommodity applications, pp. 91-107.
- [11] J. Keppo, "Pricing of bandwidth derivatives under network arbitrage condition," http://www-personal.engin.umich.edu/ keppo/BandNetwork.pdf, 2002.
- [12] J. Keppo, S. Rinaz, and G. Shah, "Network pricing with continuous uncertainties and multiple QoS classes," http://wwwersonal.engin.umich.edu/ keppo/QoSPricing.pdf, 2002.
- [13] J. Keppo, "Optimality with telecommunications network," http://wwwpersonal,engin.umich.edu/ keppo/network.pdf, 2002.
- [14] D. "Modeling the market bandwidth," Upton, http://www.statslab.cam.ac.uk/ dsu20/Papers/essay.pdf, Oct 2002.
- [15] C. Kenyon and G. Cheliotis, "Stochastic models for telecom commodity prices," Computer Networks, vol. 36, pp. 533-555, Aug 2001.
- [16] V. Firoiu, J.-Y. L. Boudec, D. Towsley, and Z.-L. Zhang, "Theories and models for internet quality of service," Proceedings of the IEEE, vol. 90, no. 9, pp. 1565-1591, Sep 2002. [Online]. Available: http://www-inst.eecs.berkeley.edu/Teresat/qosSurvey.pdf
- [17] I. Stoica, H. Zhang, S. Shenker, R. Yavatkar, D. Stephens, A. Malis, Y. Bernet, Z. Wang, F. Baker, J. Wrocławski, C. Song, and R. Wilder, "Per hop behaviors based on dynamic packet states," http://www-2.cs.cmu.edu/ istoica/DPS/draft.txt, 1999.
- [18] Z. L. Zhang, Z. Duan, and Y. T. Hou, "Virtual time reference system: A unifying scheduling framework for scalable support of guaranteed services," IEEE Journal on Selected Areas in Communication, vol. 18, no. 12, pp. 2684-2695, Dec 2000.
- [19] A. Elwalid, D. Mitra, and R. Wenworth, "A new approach for allocating buffers and bandwidth to heterogeneous, regulated traffic in ATM node, IEEE Journal of Selected Areas in Communications, vol. 13, pp. 1048-1056, 1995.
- [20] G. Huston, Internet Performance Survival Guide: QoS Strategies for Multiservice Networks. Hoboken, NJ: John Wiley, 2000.
- [21] R. Braden and et al, "Integrated services in the internet architecture: an overview," IETF Internet RFC 1633, Jun 1994.
- [22] H. Schulzrinne and et al, "RTP: A transport protocol for real-time applications," IETF Internet RFC 1889, 1997.
- [23] E. Rosen and et al, "Multiprotocol label switching architecture," IETF Internet RFC 3031, Jan 2001.
- [24] D. Awduche, A. Chiu, A. Elwalid, I. Widjaja, and X. Xiao, "Overview and principles of internet traffic engineering," IETF Internet RFC 3272, May 2002.
- [25] S. Blake, D. Black, M. Carlson, E. Davies, Z. Wang, and W. Weiss, "An architecture for differentiated services," IETF Internet RFC 2475, Dec

- [26] D. Clark and W. Feng, "Explicit allocation of best-effort packet delivery service," IEEE/ACM Transactions on Networking (TON), vol. 6, no. 4, pp. 362-373, Aug 1998.
- [27] V. Jacobson and et al, "An expedited forwarding PHB," IETF Internet RFC 2598. Jun 1999.
- [28] S. I., S. Shenker, and H. Zhang, "Core-stateless fair queueing: Achieving approximately fair bandwidth allocations in high speed networks, Proceedings of ACM SIGCOMM '98, pp. 118-130, Aug 1998.
- [29] R. Braden and et al, "Resource Reservation Protocol (rsvp) VI functional Spec," IETF Internet RFC 2205, Sep 1997.
- [30] D. Durham and et al, "The COPS (common open policy service) protocol," IETF Internet RFC 2748, Jan 2000.
- [31] S. Giordano, S. Salsano, S. V. den Berghe, B. Ventre, and D. Giannakopoulos, "Advanced QoS provisioning in the ip networks: The european premium ip projects," IEEE Communications Magazine, vol. 41, no. 1, pp. 30-36, Jan 2003.
- G. Cortese, R. Fiutem, P. Cremonese, S. D'Antonio, M. Esposito, S. P. Romano, and A. Diaconescu, "CADENUS: Creation and deployment of end-user services in premium ip-networks," IEEE Communications Magazine, vol. 41, no. 1, pp. 54-60, Jan 2003.
- T. Engel, H. Granzer, B. Koch, M. Winter, P. Sampatakos, I. Venieris, H. Hussmann, F. Ricciato, and S. Salsano, "AQUILA: Adaptive resource control for QoS using an ip-based layered architecture," IEEE Communications Magazine, vol. 41, no. 1, pp. 46-53, Jan 2003.
- [34] E. Mykoniati, C. Charalampous, P. Georgatsos, T. Damilatis, D. Goderis, P. Trimintzios, G. Pavlou, and D. Griffin, "Admission control for providing QoS in diffserv ip networks: The TEQUILA approach, IEEE Communications Magazine, vol. 41, no. 1, pp. 38-44, Jan 2003.
- [35] R. Roth, S. L. M. Champanella, R. Sabatino, N. Simar, M. Przybylski, S. Trocha, A. Liakopoulos, and A. Sevasti, "Ip QoS across multiple management domains: Practical experiences from pan-european experiments," IEEE Communications Magazine, vol. 41, no. 1, pp. 62-69, Jan 2003
- [36] R. Wilson, Nonlinear Pricing. New York, NY: Oxford University Press, Inc., 1993.
- [37] A. Gupta, S. Kalyanaraman, R. Pradhan, and M. Aboulfadl, "A spot pricing framework to enable pricing and risk management of inter-domain assured bandwidth services," Proceedings of the 2002 Winter Simulation Conference, 2002.
- R. Dolan and H. Simon, Power Pricing How Managing Price Transforms the Bottom Line. New York, NY: Free Press, 1997.
- [39] V. Paxson, "Empirically-derived analytic models of wide-area tcp con-nections," IEEEACM Transactions on Networking, vol. 2, no. 4, pp. 316-336, 1994,
- O. Cappé, E. Moulines, J.-C. Pesquet, A. Petropulu, and X. Yang, "Long-range dependence and heavy-tail modeling for teletraffic data," Jan 2002. [Online]. Available: http://www.ece.drexel.edu/CSPL/ publications/lrd.pdf
- [41] C. R. Cunha, A. Bestavros, and M. E. Crovella, "Characteristics of www client based traces," Technical Report, no. BU-CS-95-010, 1995.
 [42] NLANR, "National laboratory for applied network research,"
- http://www.nlanr.net/NA/Learn/daily.html, 2002.
- V. Paxson and S. Floyd, "Wide area traffic: The failure of poisson modeling," IEEEACM Transactions on Networking, vol. 3, no. 3, pp. 226-244 1995
- M. Crovella and A. Bestavros, "Self-similarity in world wide web traffic: Evidence and possible causes," IEEEACM Transactions on Networking, vol. 5, no. 6, pp. 835-846, 1997.
- M.E. Crovella and L. Lipsky, Self-similar network traffic and performance evaluation. New York, NY: Wiley Inter-Science Publications, 2000, ch. Simulations with Heavy-Tailed Workloads, pp. 89-100.