Performance and Robustness Analysis of an H-infinity Based Flow Controller via Simulations and Real-Time Experiments*

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February 10, 1999

Submitted to be included in an invited session on
Control Methods for Communication Networks
at the **1999 CDC**.

Abstract

In a recent work the authors an $\mathcal{H}^\infty$ based flow controller was designed for explicit rate based congestion control in high speed networks. Time delay uncertainties were taken into account when controller was designed. This paper studies computes robustness margins, e.g. largest allowable time delay, as functions of a weighting parameter used in the definition of the $\mathcal{H}^\infty$ optimization problem. Another issue discussed here is "non-fragile" (this is a recently coined term which stands for *internally robust*) implementation of the $\mathcal{H}^\infty$ controller. Time domain performance is demonstrated via simulations. Real-time experimental results will be included in the full version of the paper.

Keywords: Communication networks, Flow control, $\mathcal{H}^\infty$ control, Time delay systems.

*This work is supported in part by the National Science Foundation under grant numbers ANI-9806660, ANI-9809018, INT-9809903, by the Scientific and Technical Research Council of Turkey, and by the Korea Science and Engineering Foundation.

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Extended Summary

1 Previous Work

In a recent work of the authors an $\mathcal{H}^\infty$ based flow controller was designed for an explicit rate feedback based congestion control in high speed networks, [23]. The basic problem considered here involves a bottleneck node with n source connections as shown in Figure 1. The *queue length* at the bottleneck node is $q(t) \geq 0$, and the *flow rate* assigned to the ith source by the feedback controller is $r_i(t) \geq 0$. The maximum rate at which the ith source is able to send data is denoted by d_i, and it is called the *demand* of the ith source. The outgoing flow rate, $c(t)$, is the *channel capacity*. There are time delays in the feedforward and feedback paths between the sources and the bottleneck node.

A simple linear time invariant model is assumed for $\mathcal{H}^\infty$ controller design. Given nominal values for time delays, and upper bounds on delay uncertainty, a robustly stabilizing controller is designed. The controller is also desired to track $q_d(t)$ (reference queue length, which usually is a step-like function).

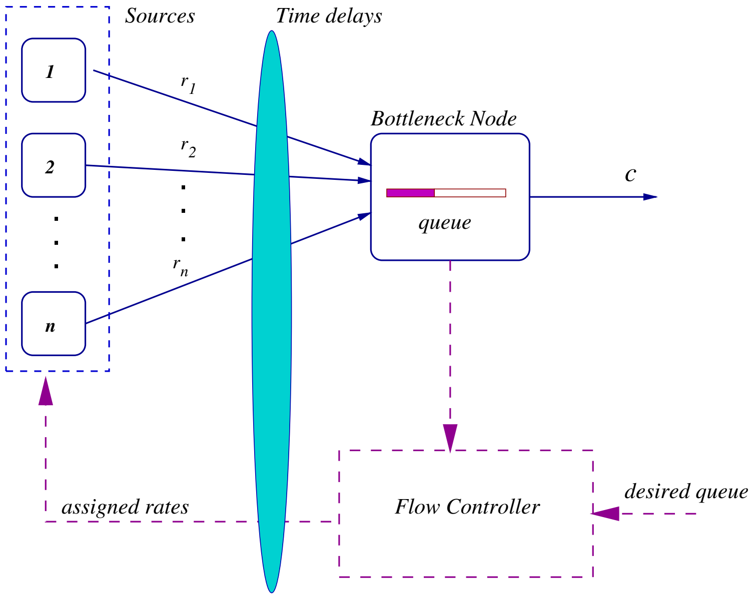

Figure 1: Flow Control Problem

The following dynamical model is considered for controller design:

$$\dot{q}(t) = \sum_{i=1}^{n} r_i(t-\tau_i) - c(t) \tag{1}$$

where τ_i is the return trip time delay between the ith source and the bottleneck node.

In order to maximize the utilization, and achieve fairness in steady state, the rates should satisfy

$$\lim_{t\to\infty} r_i(t) = c_\infty/n,$$

where $c_\infty := \lim_{t\to\infty} c(t)$.

The flow rates are restricted by $0 \leq c(t) \leq c^+$, and $0 \leq r_i(t) \leq d_i$, for all $i = 1, \dots, n$, for some upper bounds c^+ and $d_1, \dots, d_n$. The nonlinearities are ignored when $\mathcal{H}^\infty$ controller is designed, [23]. In this paper, they will be taken into account when the controller is modified, and the feedback system is simulated under different conditions.

The plant is a Multi Input Single Output (MISO) system whose transfer function is

$$P(s) = \frac{1}{s}[D_1(s) \ , \ \dots \ , \ D_n(s)],$$

where $D_i(s) = e^{-\tau_i s}$ represents the time delay in the ith channel. Let $h_i \geq 0$ be a given nominal value of τ_i, and define $\delta_i := (\tau_i - h_i)$. Suppose that $\delta_i \in (-\delta_i^+ \ , \ \delta_i^+)$, where $\delta_i^+ \geq 0$ is a known estimate of the uncertainty. The controller is a SIMO system with transfer function $K(s) = [K_1(s) \ , \ \dots \ , \ K_n(s)]^{\mathrm{T}}$.

Following the basic arguments of [23] it can be shown that for good tracking of step-like functions and good stability margins, the controller can be chosen to be in the form

$$K(s) = D_e(s)\frac{1}{nh}K_\nu(s) \tag{2}$$

where $D_e(s) = [e^{-(h-h_1)s}, \dots, e^{-(h-h_n)s}]^{\mathrm{T}}$, $h := \max\{h_1, \dots h_n\}$, (the term $D_e(s)$ is equalizing the nominal time delays in communication channels), and the scalar part of the controller $K_\nu(s)$ is determined from an $\mathcal{H}^\infty$ optimization:

$$\gamma_{\text{opt}} := \inf_{K_\nu \ \textit{stabilizing} \ P_\nu} \left\| \begin{bmatrix} W_1(1+P_\nu K_\nu)^{-1} \\ W_2 P_\nu K_\nu (1+P_\nu K_\nu)^{-1} \end{bmatrix} \right\|_\infty \tag{3}$$

with

$$P_\nu(s) = \frac{e^{-hs}}{(hs)},$$

$W_1(s) = \frac{1}{(hs)^2}$ and $W_2(s) = \rho hs$, where

$$\rho := \frac{1}{nh}\sum_{i=1}^{n} \delta_i^+ =: \frac{\delta}{h}$$

is the relative delay uncertainty. The problem data in (3) is a function of the normalized frequency $\hat{s} := hs$, so is the optimal solution $K_{\nu,\text{opt}}$.

Again in [23] it was observed that $K_{\nu,\text{opt}}$ can be computed by applying the results of [27]:

$$K_{\nu,\text{opt}}(s) = \frac{R(\hat{s})}{1 + R(\hat{s})\frac{e^{-\hat{s}}}{\hat{s}}} \left(\frac{1 - \gamma_{\text{opt}}^2 \hat{s}^4}{\gamma_{\text{opt}}^2 \hat{s}^4} \right) \tag{4}$$

where

$$R(\hat{s}) = \frac{\gamma_{\text{opt}}}{\rho} \ \frac{\hat{s}^3(\alpha - \hat{s})}{(\hat{s}+a)(\hat{s}^2 + b\hat{s} + c)(\alpha + \hat{s})}.$$

The optimal cost $\gamma_{\rm opt}$, and the other controller parameters a, b, c, α are calculated as follows: for a fixed $\gamma > 0$ let x be the unique positive root of

$$x^3 + \frac{1}{\gamma^2} x^2 - \frac{1}{\rho^4} = 0$$

and define

$$a = \frac{1}{\rho\sqrt{x}} \ , \quad c = \rho a \ , \quad b = \sqrt{2c - a^2} \ , \quad \alpha = \frac{1}{\sqrt{\gamma}} \, \frac{1 - \rho e^{\frac{1}{\sqrt{\gamma}}} (\frac{1}{\sqrt{\gamma}} + a)(\frac{1}{\gamma} + b\frac{1}{\sqrt{\gamma}} + c)}{1 + \rho e^{\frac{1}{\sqrt{\gamma}}} (\frac{1}{\sqrt{\gamma}} + a)(\frac{1}{\gamma} + b\frac{1}{\sqrt{\gamma}} + c)}$$

then $\gamma_{\rm opt}$ is the largest γ which satisfies

$$1 + R(\beta)\frac{e^{-\beta}}{\beta} = 0, \qquad \text{with} \qquad \beta = j\frac{1}{\sqrt{\gamma}}.$$

A numerical example is given below for different values of the weighting parameter ρ.

ρ	0.2	0.8	2	10
$\gamma_{\rm opt}$	1.37	2.43	3.74	8.77
$-\alpha$	0.61	0.45	0.36	0.23
a	1.73	1.10	0.82	0.49
b	1.67	1.03	0.75	0.42
$\sqrt{c}$	1.70	1.07	0.78	0.46

In the present paper following issues will be addressed.

- Stability margins will be examined for the above $\mathcal{H}^\infty$ controller for different values of the design parameter ρ. In particular largest allowable time delay (delay margin) will be determined as a function of ρ.
- The optimal controller expression (4) is quite "fragile" in the sense that there are unstable pole-zero cancellations *internally* in the controller: closed right half plane roots of $(1 - \gamma_{\rm opt}^2 \hat{s}^4)$ cancel the roots of $(1 + R(\hat{s})\frac{e^{-\hat{s}}}{\hat{s}})$ at the same points. Since the denominator is a quasi-polynomial, (i.e. polynomials of s and e^{-hs}) direct pole zero cancellation based model reduction is not possible. A different form of the same controller will be derived. In its new form the controller can be implemented in a stable fashion.
- The controller will be modified to reduce the effects of nonlinearities, and its performance and robustness analysis will be done via simulations.
- In the full version of the paper experimental results will also be presented. Current efforts are concentrated on algorithmic implementation of the controller and experiment designs. The results will be compared to some benchmark examples.

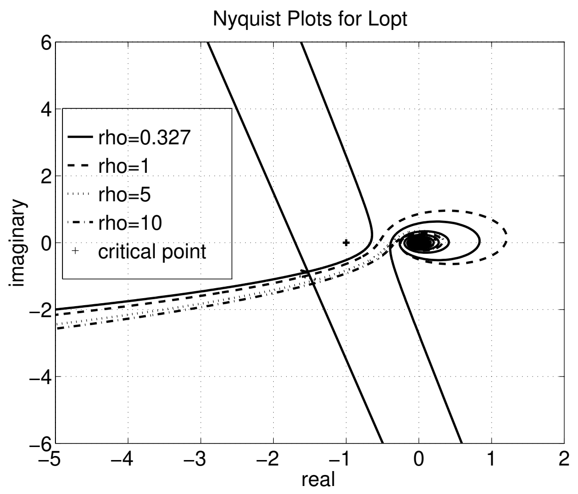

Figure 2: Nyquist Plots.

2 Stability Margins

The optimal open loop gain

$$L_{opt}(s) = P_\nu(s)K_{\nu,\mathrm{opt}}(s) = P_o(s)K_{\mathrm{opt}}(s) = \frac{R(\hat{s})\frac{e^{-\hat{s}}}{\hat{s}}}{1+R(\hat{s})\frac{e^{-\hat{s}}}{\hat{s}}}\left(\frac{1-\gamma_{\mathrm{opt}}^2\hat{s}^4}{\gamma_{\mathrm{opt}}^2\hat{s}^4}\right), \tag{5}$$

is a function of the normalized frequency $\hat{s}$. This means that in the $\mathcal{H}^\infty$ optimal design setting considered here the bandwidth of the optimal system is inversely proportional to the time delay h. The Nyquist plots for L_{opt} are shown in Figure 2 for different values of ρ. Since the plant P_ν does not have any poles in the open right half plane, and since the controller stabilizes the feedback system by design, it can be deduced that the optimal controller is unstable for $\rho \le 0.327$. The smallest value of ρ that yields a stable optimal controller is calculated to be approximately 0.33. For such time delay systems it was shown that as ρ decreases the number of encirclements of the critical point (-1,0) increases, and in the limit for $\rho = 0$ we expect to have a controller with infinitely many unstable poles, [10, 19].

For good stability robustness (i.e. $L_{\mathrm{opt}}(j\omega)$ is far from the critical point) the design parameter ρ should be large. On the other hand, if ρ is too large, time domain performance may deteriorate (this will be demonstrated later when controller implementation and simulations are discussed). The minimum distance between $(-1,0)$ and $L_{\mathrm{opt}}(j\omega)$ is

$$\eta := \inf_\omega |1 + L_{\mathrm{opt}}(j\omega)| = \left(\|(1+P_oK_{\mathrm{opt}})^{-1}\|_\infty\right)^{-1}$$

The plot of ρ versus η is shown in Figure 3.

By the definition of two block problem (3)

$$\|\frac{\rho}{\gamma_{\mathrm{opt}}}K_{\nu,\mathrm{opt}}(1+P_\nu K_{\nu,\mathrm{opt}})^{-1}\| \le 1,$$

so a lower bound of the maximal allowable relative time delay uncertainty $(h_{\max} - h)/h$ is $\rho/\gamma_{\mathrm{opt}}$. It is well known that, for a stable feedback system with open loop transfer function $L_o(s)$, the delay

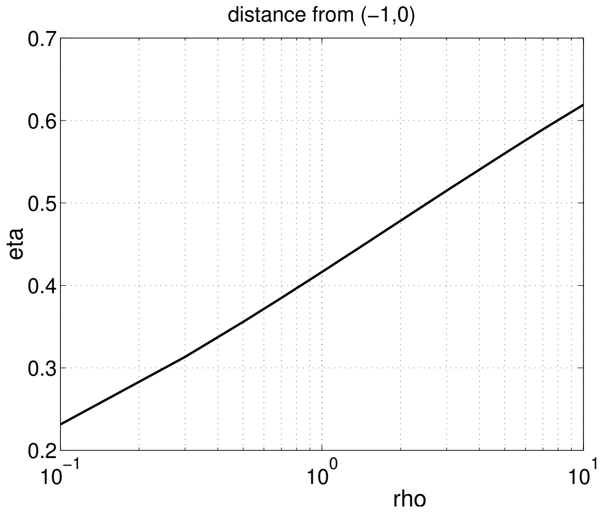

Figure 3: A Robustness Measure: η versus ρ.

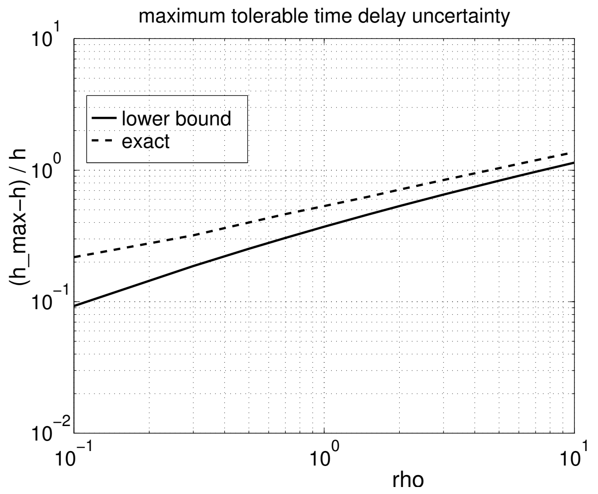

Figure 4: Largest Allowable Time Delay Uncertainty.

margin (i.e. the largest $h' > 0$ for which the feedback system remains stable when the open loop transfer function is perturbed to $e^{-h's}L_o(s)$) is given by $h' = \phi/\omega_c$, where ϕ is the phase margin in radians, and ω_c, in radians/sec., is the gain crossover frequency (i.e. $|L_o(j\omega_c)| = 1$) determined from the Bode or Nyquist plots of $L_o(s)$. Hence the quantity $(h_{\max} - h)/h$ can be determined exactly from $L_{\rm opt}(j\omega)$; Figure 4 shows the lower bound and the exact value of the maximum allowable time delay uncertainty as a function of ρ.

3 Controller Implementation

The controller expression given by (4) cannot be implemented in a stable manner, because there are unstable pole-zero cancellations internally in the controller: some of the the unstable roots of $(1 + R(\hat{s})\frac{e^{-\hat{s}}}{\hat{s}})$ are canceled by the roots of $(1 - \gamma_{\rm opt}^2 \hat{s}^4)$. But a simple algebra shows that the optimal

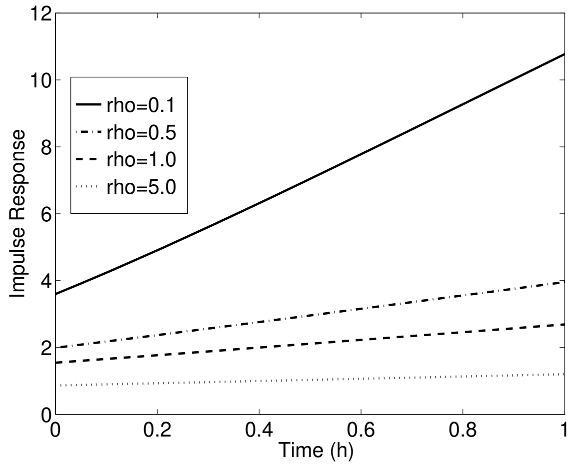

Figure 5: Impulse Response of $F(\hat{s})$.

controller (4) can be re-written as

$$K_{\nu,\mathrm{opt}} = \frac{\gamma_{\mathrm{opt}}}{\rho} \frac{(\hat{s}-\alpha)}{\hat{s}} \frac{1}{1+F(\hat{s})} \tag{6}$$

where

$$F(\hat{s}) = \frac{(\hat{s}+a)(\hat{s}^2+b\hat{s}+c)(\hat{s}+\alpha) - (\hat{s}^4-\gamma_{\mathrm{opt}}^{-2})}{(\hat{s}^4-\gamma_{\mathrm{opt}}^{-2})} - \frac{\gamma_{\mathrm{opt}}\rho^{-1}\hat{s}^2(\hat{s}-\alpha)e^{-\hat{s}}}{(\hat{s}^4-\gamma_{\mathrm{opt}}^{-2})}. \tag{7}$$

The interpolation conditions from which controller parameters $\{\gamma_{\mathrm{opt}}, \alpha, a, b, c\}$ are determined, [27], imply that $F(\hat{s})$ and $e^{-\hat{s}}F(-\hat{s})$ are stable. Therefore all of the four poles of $F(\hat{s})$ are canceled internally in (7). In terms of state space realizations of the two additive terms of (7), a realization of $F(\hat{s})$ can be given in the form

$$F(\hat{s}) = C(e^{-\hat{s}}I - e^{-A})(\hat{s}I-A)^{-1}B \tag{8}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \gamma_{\mathrm{opt}}^{-2} & 0 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad C = \frac{\gamma_{\mathrm{opt}}}{\rho} \begin{bmatrix} 0 & 0 & \alpha & -1 \end{bmatrix}.$$

The expression (8) demonstrates that the impulse response of $F(\hat{s})$ is restricted to the time interval $[0\ ,\ 1]$. A similar $\mathcal{H}^\infty$ controller structure was obtained in [21] for this type of delay systems. The impulse response of $F(\hat{s})$ is shown in Figure 5 for several different values of ρ.

Hence a "non-fragile" (internally robust) digital implementation of the controller (6) includes a PI term which is cascaded with a feedback block containing an FIR filter. The length of the FIR filter is h/T_s, where T_s is the sampling period. If h is large with respect to T_s then the order of the FIR filter, hence the order of the controller is large. In the context of control of ATM networks, fragility and robustness issues were discussed in [6].

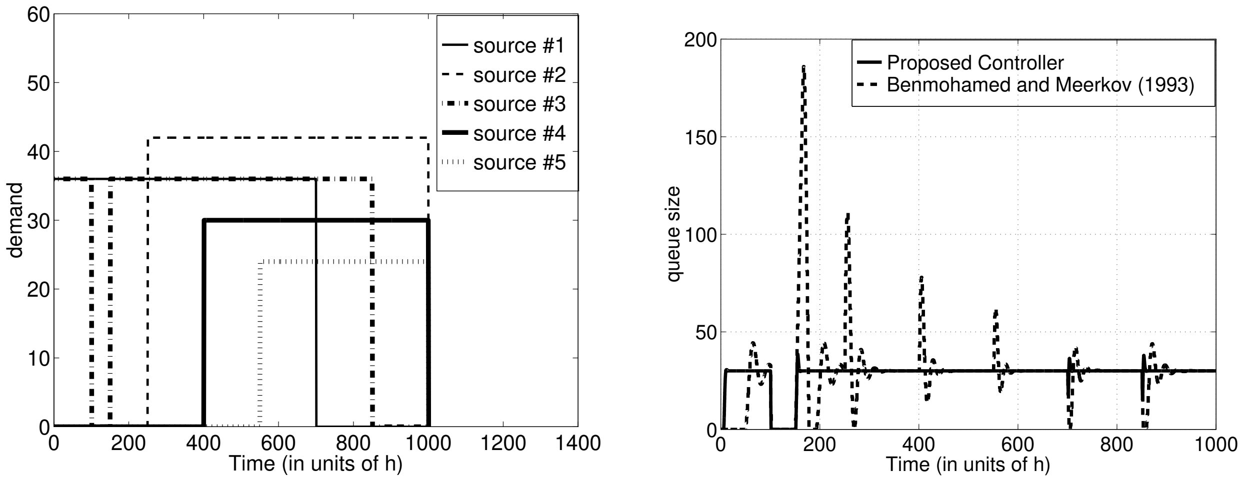

Figure 6: Source Connection Times and Demands, and Resulting Queue Size.

4 Simulation Studies

The controller (6) is derived from $\mathcal{H}^\infty$ optimization, so nonlinearities in the actual system are not taken into account. Recall that the maximal data rate (demand) for the ith source is d_i, and in steady state the controller assigns equal rates, c_∞/n, to different sources. If the assigned rate is higher than d_i for the ith source, then the rate command for this particular source should be saturated at d_i. Suppose that $(d_1(t) + \ldots + d_n(t)) \geq c(t)$ for all t, i.e. total demand is higher than the capacity of the bottleneck link. In this case, although some sources may be saturated, by distributing the remaining total rate allocation to sources with higher demands, overall operation can be kept at the linear range. Another key assumption made in the controller design is that the number of active sources, n, and corresponding d_i's are known. This is a reasonable assumption in the sense that when sources sign-in and sign-off from the network this type of information is made available to the flow controller. Accordingly, in the SIMULINK based implementations, modifications are made on the basic linear controller (6) to include logical blocks to perform saturation checks, rate allocations to different sources, time delay equalization units, and a counter for the number of active sources.

In order to reduce the effect of the saturation type of nonlinerity in the queue size, the integrator in (6) is implemented with an anti-windup logic. In the SIMULINK based implementation used for time domain results to be presented here, the integration, at the PI term of the controller, is stopped if all sources become saturated, and then the integrator is reset to zero when at least one of the sources becomes unsaturated; that prevents excessive overshoot in the queue size.

The benchmark example selected for simulations is from [5], where five different sources are sending data through a bottleneck node at different time instants with different demands. Considering Table II of [5] the roundtrip delays for sources 1, 2, 3, 4, and 5 are taken to be 0, 0, $20\tau_s$, $40\tau_s$, and $60\tau_s$, respectively, where τ_s is the transmission time of a single packet. Thus, the maximum roundtrip delay is $h := 60\tau_s$. The computational time delays are neglected. Following the example given in [5] the queue clearing rate is chosen as $c(t) = 60$ packets per h unit of time, and desired queue is taken to be $q_d = 30$ packets. Figure 6 shows a comparison of the response corresponding to the controller proposed here (for $\rho = 2$) with the benchmark response given in [5]. The benchmark controller in [5] updates its output once every h units of time, whereas in the simulations performed for the controller developed here the controller output is updated 20 times per h units of time, i.e. $T_s = h/20$.

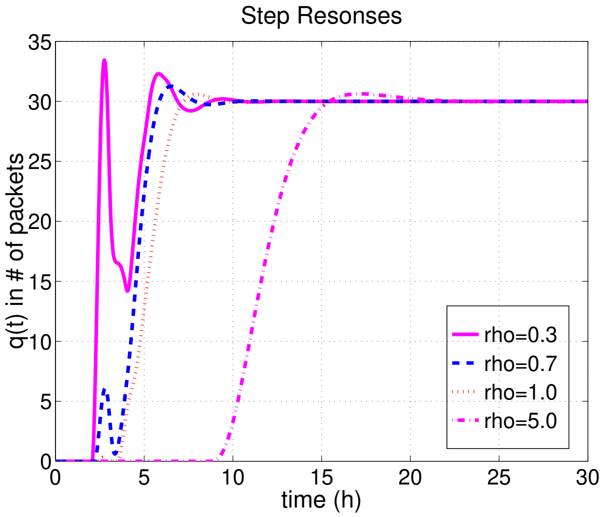

Figure 7: Step Responses.

The step response ($q(t)$ versus t) is obtained by choosing a sufficiently large fixed value for the total demand. For example, if the simulation, shown in Figure 6, is stopped at time instant $30h$, then it simply gives the response to demands of first and third sources (total demand here is $(36+36) > 60 = c(t)$, and because of delay equalization for all five sources, the total time delay is still $h = 60\tau_s$). For different values of ρ, the step response is as shown in Figure 7. Note that larger the value of ρ, slower the step response. The basic trade-off in selection of ρ is now clear: if ρ is too large the response is slow, but if it is too small the stability robustness margin is small. By looking at the step responses and the delay margin plots it is determined that $\rho = 2$ is a reasonably good choice.

Figure 8 shows the step responses corresponding to the cases where the controller uses h as the time delay, but actual time delay is different, say τ. It can be seen that the system enters into a limit cycle when actual time delay is $\tau = 2h$ i.e. it is 100% more than the value of the time delay used in the controller. This not surprising since for the value of $\rho = 2.0$ Figure 4 shows that the largest allowable time delay uncertainty is about 70%, i.e. the system can tolerate $\tau = 1.7h$ but $\tau = 2h$ leads to instability.

The step response corresponding to a time-varying delay uncertainty is shown in Figure 9. In this case all five sources are made active (the number of active sources detected by the controller as a function of time is also shown in the figure), and additive delay perturbations in the form $h(0.5 + \sin(\frac{2\pi}{10h}t))$ are injected to channels 4 and 5 (in these channels nominal time delays are $\frac{2}{3}h$ and h, respectively). The additive delay perturbations in channels 4 and 5 take values between $-0.5h$ and $1.5h$, with and average of $0.5h$. Note that the nominal delay in other channels are less than $0.5h$, to avoid negative time delays (non-causal situations) the time delay perturbations are not introduced in the first three channels.

The effect of variations in $c(t)$ is demonstrated in Figure 11, where $q_d(t) = 30$ packets is fixed and $c(t)$ is taken to be a time varying function shown in the Figure 10. It is observed that fast sinusoidal variations in $c(t)$ lead to relatively small variations in $q(t)$. The effects of sudden "step-like" changes are more significant, but they are transient.

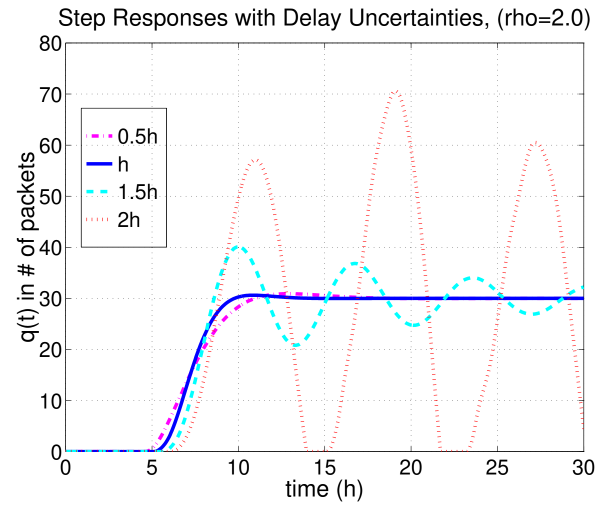

Figure 8: Step Response with Uncertain Delays, $\rho = 2.0$.

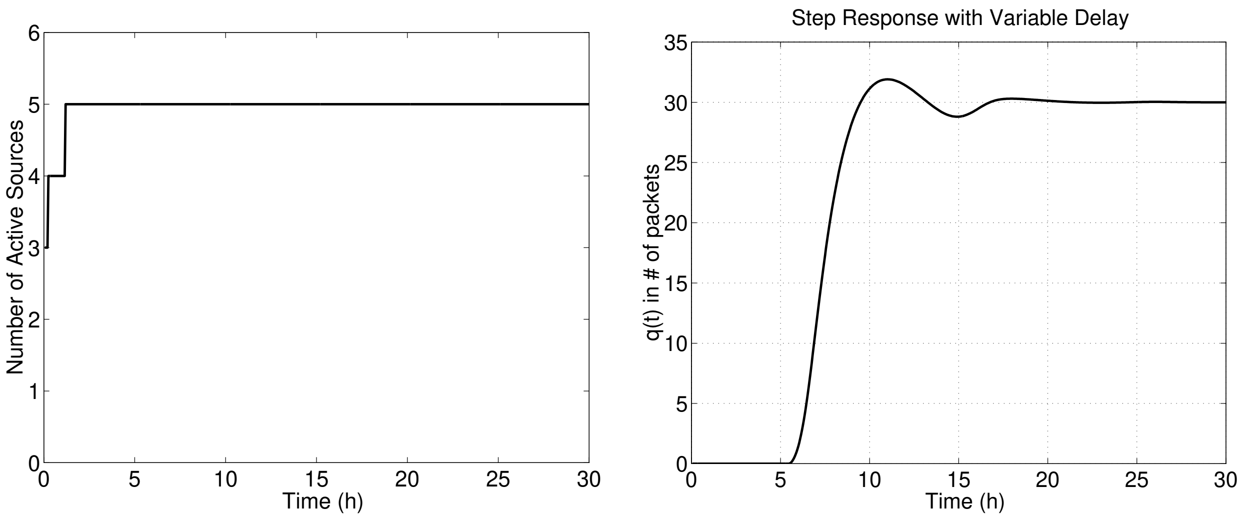

Figure 9: Step Response with Time Varying Delays, $\rho = 2.0$.

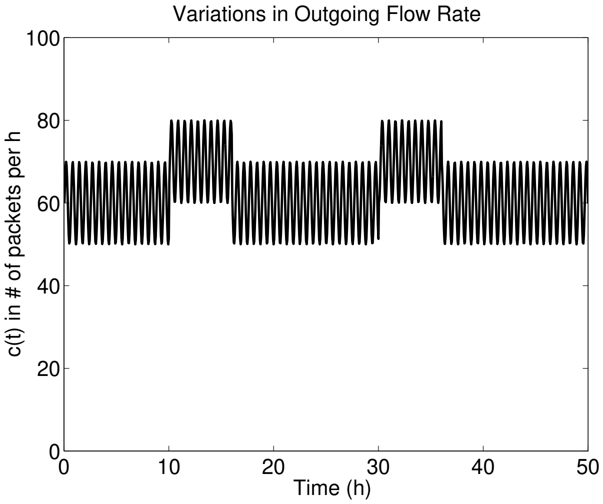

Figure 10: Variations in $c(t)$.

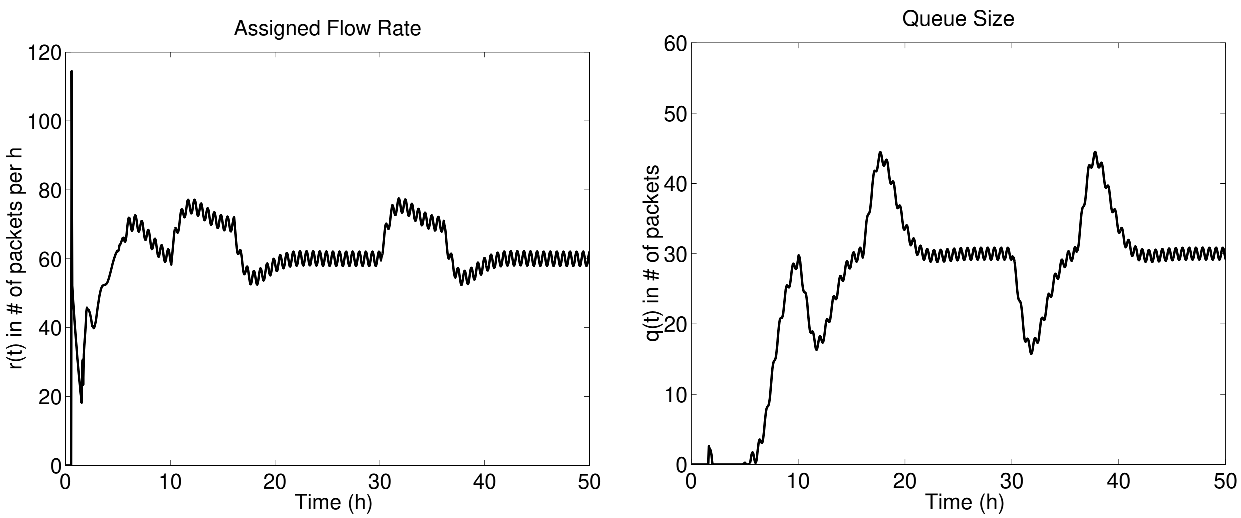

Figure 11: Response with Non-Constant $c(t)$, $\rho = 2.0$.

5 Conclusions

Robustness issues related to the implementation of the $\mathcal{H}^\infty$ controller designed in [23] are discussed here. It is shown that the optimal controller can be implemented in an internally robust (non-fragile) fashion: in the form of a PI term cascaded with a feedback term involving a FIR filter. This controller robustly stabilizes the feedback system in the presence of delay uncertainties. Largest allowable time delays, and other stability robustness measures are illustrated. Minor nonlinear modifications are made in the SIMULINK based realization of the controller. Simulations show that the design framework considered here allows fluctuations in the capacity $c(t)$, as well as in the time delays. In the full version of the paper real-time experimental results will also be presented: currently the controller is being implemented on a physical ATM switch.

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