On the Delay Violation Probability for Earliest Deadline First Schedulers

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***Abstract*— Recently, an end-to-end statistical quality of service architecture using Earliest Deadline First (EDF) schedulers was demonstrated. In this paper we examine an important aspect of this proposal, namely, the probability of violation of delay assurances. An approach to obtain the violation probability is to use a worst-case characterization of input traffic. Existing work uses a traffic profile which is not proved to be worst-case.**

We take advantage of a recent work deriving an extremal shape-controlled traffic profile. We derive the key results required to obtain the delay violation probability using this provably worst-case traffic profile. We then compare the resulting expressions with existing work. This leads to more accurate estimation of the admissible region and end-to-end delay assurance.

I. Introduction

Assured delay services are required for a variety of modern applications involving video and voice traffic. To obtain end-to-end delay assurances one could adopt specialized schedulers in the network which are aware of delay constraints of incident flows. The Generalized Processor Sharing [PaGa93], [PaGa94] and Earliest Deadline First [FeVe90] scheduling disciplines are among the popular choices. A framework for end-to-end statistical delay assurances using EDF schedulers was proposed by Sivaraman and Chiussi [SiCh00].

An important aspect of the end-to-end QoS framework is that of estimating the probability of delay violation at each node of the network. One approach to obtain a bound for the probability of delay violation is to employ *Statistical Service Envelopes*. Qiu and Knightly [QiKn99] have demonstrated that using stochastic bounding variables, the probability of delay violation for various scheduling disciplines can be bounded using a Gaussian approximation.

An alternative approach is to estimate the the *unfinished work* in the system (fluid equivalent of queue length) using the Benes formula as discussed Sivaraman and Chiussi [SiCh00]. In order to obtain an approximate evaluation of the Benes formula, we require a precise description of the traffic characteristics. For a conservative approach to network design, one would consider the worst-case traffic descriptions (extremal traffic descriptions). Previous frameworks [ElMiWe95], [SiCh00] have employed an on-off traffic description which is not necessarily extremal. Recent work by Kesidis and Konstantopoulus [KeKo00] has demonstrated methods to obtain verifiably extremal traffic descriptions.

We first obtain the extremal traffic description using the methods in [KeKo00]. We then use this description to derive the quantities involved in the Benes Formula. Finally, we discuss how to utilize the results obtained and their significance. We compare the methods employed in the process with existing work.

The contributions of this paper are in deriving the expressions for quantities involved in the approximate evaluation of the Benes formula, adopting a provably extremal traffic description for the purpose, and hence facilitating accurate bounds for delay violation probability in EDF schedulers.

The rest of the paper is structured as follows. In Section III-A we discuss the Benes approach to calculating the unfinished work in a system. We then examine an extremal traffic description in Section IV-A and use it to obtain improved delay bounds. The results so obtained are compared with existing work in Section V and the conclusions are noted in Section VI.

II. Notation and Assumptions

We briefly note the symbols used in the rest of the paper. A source is characterized by the triple (p, ρ, σ), where is p is the peak allowable rate, and (ρ, σ) indicates the rate and bucket depth of a leaky-bucket. The unfinished work in the system at time t is denoted by V_t and its complementary distribution is denoted by $\nu(x)$. The quantity C indicates the capacity of the server. b

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is such that the source traffic maximizes the probability that a bit suffers a delay greater than b/C. It is assumed that $\rho < C < p$ where ρ and p indicate the shaper rate and peak rate for the aggregate traffic.

The i^{th} flow from the j^{th} class is represented as (j, i). The parameters of a flow are represented as (p_j, ρ_j, σ_j).

III. Mathematical Formulation

As discussed earlier, the delay violation probability can be estimated by considering the maximum unfinished work remaining in the system. In order to compute unfinished work, we employ the Benes formula (Section III-A). Evaluating the formula is hard and needs certain approximations which are briefly mentioned in Section III-B.

A. Benes Approach

Let $W(t), t \geq 0$ denote the amount of work arriving to the system in the interval $[-t, 0)$ and let V_t be the amount of unfinished work in the system at time $-t$. Define $X(t) = W(t) - t, t \geq 0$ to be the excess work arriving in $[-t, 0)$. Let $\nu(x)$ be the complementary distribution of V_0, i.e., $\nu(x) = P\{V_0 > x\}$. Then Benes formula can be used to obtain $\nu(x)$ as follows:

$$\nu(x) = \int_{u>0} P\{X(u) \geq x > X(u+du) \; and \, V_u = 0\} \quad (1)$$

$$\leq \int_{u>0} P\{X(u) \geq x > X(u+du)\} \quad (2)$$

The upper bound given by Equation (2) is often both reasonably accurate and numerically calculable. Now the probability of delay violation can be calculated as in [SiCh00]:

$$P_{vio} = \frac{1}{\rho} \lim_{x \to 0} \nu(x) \quad (3)$$

In order to evaluate Equation (3), we need a characterization of the input process. Instead of using an exact description, it is easier to obtain a worst-case description of the traffic. We shall examine one such description in the succeeding section.

B. Evaluating the Benes formula

In order to evaluate the Benes formula, we employ a construction detailed by Norros et al [NoRoSi91] and, Sivaraman and Chiussi [SiCh00]. We present here only the notation and relevant results and refer the readers to aforementioned papers.

Let Λ_t denote the arrival rate at time $-t$, and let $A(t) = \int_{-t}^{0} \Lambda_t \, dt, (t \geq 0)$ be the total amount of work arriving in the interval $[-t, 0)$. All sources (j, i) with $d_j > t$ do not contribute towards Λ_t or $A(t)$. Let V_t be the work still in the system at time $-t$ and let $X(t) = A(t) - Ct$ denote the excess work arriving in $[-t, 0)$. where C denotes the link rate. Denote $\phi_t(w, \lambda)$ denote the joint density of $A(t)$ and Λ_t:

$$\phi_t(w, \lambda) = \frac{d}{dw} P\{A(t) \leq w, \Lambda_t = \lambda\}$$

Then, it can be shown that,

$$\nu(x) \leq \int_{u>0} \sum_{0 \leq \lambda < C} (C - \lambda) \phi_u(x + Cu, \lambda) du \quad (4)$$

Using a shifted normal approximation, $\phi_t(w, \lambda)$ can be calculated as:

$$\phi_t(w, \lambda) = \frac{e^{sw} \phi_t^*(s, \lambda)}{\sqrt{2\pi} \sigma_t} \quad (5)$$

$where,$

$$\sigma_t = \sum_{1 \leq j \leq m} \left[l_j \left(\frac{d^2 \alpha_j^*}{ds^2} / \alpha_j^* - \left(\frac{d\alpha_j^*}{ds} / \alpha_j^* \right)^2 \right) + (k_j - l_j) \left(\frac{d^2 \beta_j^*}{ds^2} / \beta_j^* - \left(\frac{d\beta_j^*}{ds} / \beta_j^* \right)^2 \right) \right] \quad (6)$$

The value of s in Equation (5) is obtained as a solution to the equation:

$$w = -\sum_{j=1}^{m} \left(l_j \frac{d\alpha_j^*}{ds} / \alpha_j^* + (k_j - l_j) \frac{d\beta_j^*}{ds} / \beta_j^* \right)$$

The quantities α_j^*, β_j^* are the laplace transforms of $\alpha_j(t, w)$ and $\beta_j(t, w)$ with respect to w. Further, $\alpha_j(t, w), \beta_j(t, w), l_j, k_j$ are defined as below.

$$m = max\{j : d_j < t\} \quad (7)$$

$$l_1, l_2, \ldots l_m : \sum_{j=1}^{m} l_j p_j = \lambda \quad (8)$$

$$\alpha_j(t, w) = \frac{d}{dw} P\{A_{ji}(t) \leq w \; and \, (j, i) \, ON \, at - t\} \quad (9)$$

$$\beta_j(t, w) = \frac{d}{dw} P\{A_{ji}(t) \leq w \; and \, (j, i) \, OFF \, at - t\} \quad (10)$$

IV. Delay bounds for an Extremal Traffic profile

Given a worst-case traffic profile, we can derive the quantities α^* and β^* so that the Benes formula can be evaluated. We shall first present such a traffic description in Section IV-A following the results in [KeKo00]. We shall then employ this description to obtain the required quantities in Section IV-B.

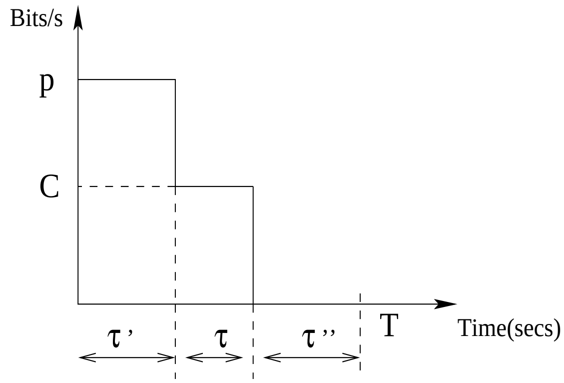

Fig. 1. An Extremal Traffic Profile

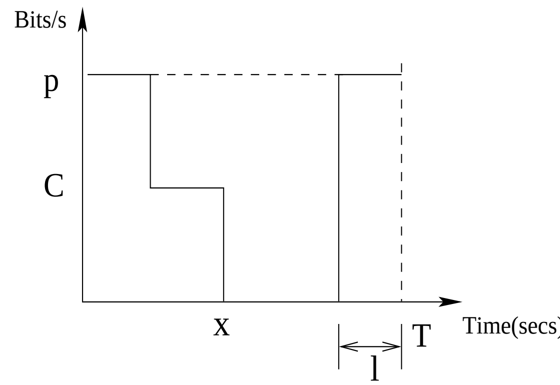

Fig. 2. A Flow in ON period with a random phase lag

A. An extremal traffic profile

Assuming the arrival process $A(t)$ to be a stationary non-decreasing stochastic process, and assuming the capacity of the server to be C, we can define the queue occupancy at time t to be:

$$Q_t = \sup_{\{-\infty < s \leq t\}} \{A(s,t] - C(t-s)\} \quad (11)$$

Then define the performance measure ψ as:

$$\psi(A) = P(Q_0 \geq b) \quad (12)$$

$$\psi^* = \sup_{All\ A} \psi(A) \quad (13)$$

The value A^* which achieves ψ^* given in Equation (13) represents an extremal traffic description.

Considering (p, σ, ρ) shaping (a peak rate of p and a leaky-bucket regulator with parameters (σ, ρ)) the authors in [KeKo00] obtain a closed form for A^*. It was shown that such a traffic process satisfies the following properties.

1) A^* is periodic.
2) Q^* should be zero at the beginning of every period, where Q^* is obtained by using Equation (11) for A^*.
3) A^* starts a period by transmitting at the peak rate p for an interval of length τ_0'.
4) After that, A^* transmits at rate C, for an interval of length τ_0.
5) Then, A^* switches off, for an interval of length τ_0''.

Here the intervals $\tau_0, \tau_0', \tau_0''$ are defined as:

$$\tau_0' = \frac{b}{p-C},\ \tau_0 = \frac{\sigma - \beta^{-1}b}{C-\rho},\ \tau_0'' = \frac{\sigma}{\rho} \quad (14)$$

where β is given by:

$$\beta = \frac{p-C}{p-\rho}$$

B. Using the Extremal traffic profile

As shown in Figure (1), an extremal leaky-bucket source transmits at the peak rate for a period τ', then at the rate C for a period τ and switches off for a period τ''. As discussed in Section IV-A the values of τ, τ' and τ'' can be expressed in terms of the leaky-bucket parameters. Further the extremal source is periodic with a period $T = \tau + \tau' + \tau''$. In order to evaluate the Benes formula we observed in Section III-B that we need to calculate the laplace transforms α^* and β^*. In the following paragraphs, we outline the method to obtain these quantities.

The flows incident at the server conform to the extremal specification as described. However they are allowed to have a random phase. Thus at any time t there will be a set of sources that are at some point in their ON period, while the rest are in some point in their OFF period. Obviously, over a duration of one period (T), the amount of data offered by each flow is the same. Thus for the rest of the discussion, we consider only the duration in the current period. Till time t there should be $\lfloor t/T \rfloor$ periods. Thus we obtain the position at the current period by deducting from t a quantity equal to $T * \lfloor t/T \rfloor$. Indicate this quantity as x_j.

Consider a flow, with a random phase lag, that is assumed to have started in an ON period at time 0. A possible position for x_j is indicated as x in Figure (2). The contribution of this flow till the point x is easily calculated in terms of the "lag" l (indicated in the figure). Thus the probability that the flow contributes a particular amount of work to the system is directly related to the lag that it features with respect to a flow whose ON period starts at 0. For example, in Figure (2), we have the contribution of the flow as $p(\tau' - l)^+ + C[x_j - (\tau' - l)^+]$. We exploit this property and the fact that the phases are uniformly distributed over the period T to calculate the quantities α^* and β^*.

In the following equations C_i refers to the share of the capacity for flow i. One way to calculate this would be in inverse proportion of the the delay allocation for the class. Thus the share of class j would be:

$$\frac{1/d_j}{\sum_k 1/d_k} C$$

Then, within the class the share of each flow can be equal. With this convention, define the following quantities:

$$\begin{aligned}
r &= \left\lfloor \frac{t}{T} \right\rfloor (C_j \tau + p\tau') \\
x_j &= t - T \left\lfloor \frac{t}{T} \right\rfloor \\
l_0 &= (\tau' - x_j)^+ \\
l_1 &= (\tau + \tau' - x_j)^+ \\
l_2 &= \tau + \tau' - (x_j - \tau'')^+ \\
l_3 &= \tau - (x_j - \tau' - \tau'')^+ \\
l_4 &= \tau + \tau' \\
a_0 &= r + x_j p \\
a_1 &= r + p(\tau' - l_1)^+ + C_j[x_j - (\tau' - l_1)^+] \\
a_2 &= r + p(\tau' - l_1)^+ + C_j[x_j - \tau'' - (\tau' - l_1)^+] \\
a_3 &= r + p\tau' + C_j(\tau' - l_2) \\
a_4 &= r + p\tau' + C_j(x_j - \tau' - \tau'')^+
\end{aligned}$$

With these quantities we can now express the laplace transforms α^* as below.

$$\begin{aligned}
\alpha^*(s,t) &= \frac{l_0}{T} e^{-sa_0} + \frac{l_1 - l_0}{Ts(a_1 - a_0)}[e^{-sa_0} - e^{-sa_1}] \\
&\quad + \frac{l_2 - l_1}{Ts(a_2 - a_1)}[e^{-sa_1} - e^{-sa_2}] \\
&\quad + \frac{l_3 - l_2}{Ts(a_3 - a_2)}[e^{-sa_2} - e^{-sa_3}] \\
&\quad + \frac{l_4 - l_3}{T} e^{-sa_4} \qquad (15)
\end{aligned}$$

With a flow beginning in the OFF period we apply a similar method. In order to obtain β^* we define the following quantities.

$$\begin{aligned}
l_0 &= (\tau'' - x_j)^+ \\
l_1 &= \tau'' - (x_j - \tau')^+ \\
l_2 &= \tau'' - (x_j - \tau' - \tau)^+ \\
a_0 &= r + p\tau' \\
a_1 &= r + p\tau' + C_j \tau
\end{aligned}$$

We now express β^* in terms of the above quantities.

$$\begin{aligned}
\beta^*(s,t) &= \frac{l_0}{T} + \frac{l_1 - l_0}{Tsa_0}[1 - e^{-sa_0}] \\
&\quad + \frac{l_2 - l_1}{Ts(a_1 - a_0)}[e^{-sa_0} - e^{-sa_1}] \\
&\quad + \frac{\tau'' - l_2}{T} e^{-sa_1} \qquad (16)
\end{aligned}$$

We now have in Equations (15,16) the required expressions to compute the Laplace transform $\phi_t^*(s,\lambda)$.

V. Discussion

We briefly discuss how to use the results and compare with previous work.

A. Computing the probabilities

In order to obtain the final violation probabilities, the integral in Equation (4) is numerically computed with $x \to 0$. This requires that Equation (5) be evaluated. The value of s in in Equation (5) is obtained by noting that w is equated to be the mean of the shifted normal distribution [NoRoSi91]. This requires the Laplace transforms α^* and β^*, so also the computation of $\phi_t^*(s,\lambda)$. Since the expressions for both α^* and β^* are in simple closed form, it is straightforward to compute the probabilities. However the time complexity of the computation is high.

B. Comparison

Previously [LoZhoTo97], [ElMiWe95], [SiCh00] the "worst-case" traffic profile considered was of the following form. The source switches on for a period of $T^o = \frac{\sigma}{p-\rho}$ and transmits at the peak rate p. Then the source switches off and stays that way for a period $T^f = \frac{\sigma}{\rho}$. This description yields a simpler form for the transforms we require. However, as mentioned in [Do95], this description need not always be the worst-case. Recently, the authors in [RaReRo98] note that a simple on/off process does not lead to a worst-case scenario.

Also the traffic profile considered in this paper is one solution to Equation (13). Thus there might be other descriptions which maximize the same overflow probability or delay assurance [KeKo00].

VI. Conclusions

In this paper we examined the probability of violation of delay assurances with EDF schedulers. In order to employ the Benes approach to find the maximum unfinished work in the system, we used a provably worst-case traffic profile. We then derived the necessary quantities to use this traffic profile in calculating the new delay bound.

The significance of the results is that the delay violation probabilities represent the worst possible traffic scenario. This facilitates accurate design of networks with EDF schedulers requiring worst-case delay violation bounds.

REFERENCES

[BeGuRo90] B. Bensaou, J. Guibert, J. Roberts, "Fluid Queueing Models for a Superposition of On/Off Sources," ITC Broadband Seminar, Morristown, October 1990

[Do95] B.T. Doshi, "Deterministic Rule Based Traffoc Descriptors for Broadband ISDN: Worst Case Behaviour and its Impact on Connection Acceptance Controls," International Journal of Communication Systems, March-April 1995

[ElMiWe95] A. Elwalid, D. Mitra, R.H. Wentworth, "A New Approach for Allocating Buffers and Bandwidth to Heterogeneous, Regulated Traffic in an ATM Node," JSAC, August 1995

[FeVe90] D. Ferrari, D. Verma, "A Scheme for Real-time Channel Establishment in Wide-area Networks," JSAC, April 1990

[KeKo00] G. Kesidis, T. Konstantopoulos, "Extremal Shape-Controlled Traffic Patterns in High-Speed Networks," Trans. on Communications, Vol 48, No. 5, May 2000

[LoZhoTo97] F. LoPresti, Z. Zhang, D. Towsley, J. Kurose, "Source Time-scale and Optimal Buffer/Bandwidth Trade-off for Regulated Traffic in an ATM Node,"INFOCOM 97

[NoRoSi91] I. Norros, J.W. Roberts, A. Simonian, J.T. Virtamo, "The Superposition of Variable Bit Rate Sources in an ATM Multiplexer," JSAC, 9(3):378-387, April 1991

[PaGa93] A. Parekh, R. Gallager, "A Generalized Processor Sharing Approach to Flow Control in Integrated Services Networks: The Single Node Case," IEEE/ACM Trans. on Networking, June 1993

[PaGa94] A. Parekh, R. Gallager, "A Generalized Processor Sharing Approach to Flow Control in Integrated Services Networks: The Multiple Node Case," IEEE/ACM Trans. on Networking, April 1994

[QiKn99] J. Qiu, E.D. Knightly, "Inter-Class Resource Sharing using Statistical Service Envelopes," INFOCOM 1999

[RaReRo98] S. Rajagopal, M. Reisslein, K.W. Ross, "Packet Multiplexers with Adversarial Regulated Traffic," INFOCOM 1998

[SiGu95] A. Simonian, J. Guibert, "Large Deviations Approximation for Fluid Queues Fed by a Large Number of On/Off Sources," JSAC, August 1995

[SiCh99] V. Sivaraman, F.M. Chiussi, "Statistical Analysis of Delay Bound Violations at an Earliest Deadline First (EDF) Scheduler," Performance Evaluation, 36(1):457-470, 1999

[SiCh00] V. Sivaraman, F.M. Chiussi, "Providing End-to-end Statistical Delay Guarantees with Earliest Deadline First Scheduling and Per-hop Traffic Shaping," INFOCOM 2000les