PERGAMON

Automatica 38 (2002) 917–928

automatica

www.elsevier.com/locate/automatica

Rate-based flow controllers for communication networks in the presence of uncertain time-varying multiple time-delays ☆

Pierre-François Quet[a], Banu Ataşlar[b], Altuğ İftar[b,*], Hitay Özbay[a], Shivkumar Kalyanaraman[c], Taesam Kang[d]

[a] *Department of Electrical Engineering, The Ohio State University, Columbus, OH 43210, USA*
[b] *Department of Electrical and Electronics Engineering, Anadolu University, Eskişehir 26470, Turkey*
[c] *Department of Electrical, Computer, and Systems Engineering, Rensselaer Polytechnic Institute, Troy, NY 12180, USA*
[d] *Department of Aerospace Engineering, Konkuk University, # 1 Whayang-dong, Gwangjin-ku, Seoul 143-701, South Korea*

Received 10 July 2000; received in revised form 24 July 2001; accepted 2 November 2001

Abstract

An $\mathscr{H}^\infty$ based robust controller is designed for a rate-feedback flow-control problem in single-bottleneck communication networks. The controller guarantees stability robustness to uncertain time-varying multiple time-delays in different channels. It also brings the queue length at the bottleneck node to the desired steady-state value asymptotically and satisfies a weighted fairness condition. Lower bounds for stability margins for uncertainty in the time-delays and for the rate of change of the time-delays are derived. A number of simulations are included to demonstrate the time-domain performance of the controller. Trade offs between robustness and time-domain performance are also discussed. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: Communication networks; Flow control; $\mathscr{H}^\infty$ control; Time-delay systems; Uncertain time-varying multiple time-delays

1. Introduction

High speed data communication networks require resource management methods in order to provide good quality of service to its users. One typical resource management tool is flow control, which is aimed at avoiding traffic congestion by regulating the rate of data packets sent from the sources. This problem has been studied widely in computer networks and communications literature, see for example Bonomi and Fendick (1995), Jain (1996), Parekh and Gallager (1994), Ramakrishnan and Newman (1995) and their references.

Feedback schemes used for flow control may be classified into two groups: "rate-based" and "window-based" (also called "credit-based") flow control. The rate-based control with explicit feedback is chosen as the standard flow control scheme in asynchronous transfer mode (ATM) switching networks, by the ATM forum (ATM Forum Traffic Management, 1996). The window-based flow control with loss-based or bit-based feedback is popular in end-to-end flow control in the Internet (e.g., TCP (Jacobson, 1988)), though rate-based schemes are also being proposed recently (Floyd, Handley, Padhye, & Widmer, 2000).

All congestion control frameworks have three main components implemented at the source end-system, switches (or routers) and destination end-systems. In the case of ATM available bit rate (ABR) service (ATM Forum Traffic Management, 1996), the sources send a control cell once every N packets (called "cells") which can be used by switches to convey feedback. The control cells travel to the destination and are returned to the source in the same path. Feedback signal may be in the form of a single bit or an explicit rate

☆ This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor R. Srikant under the direction of Editor Tamer Basar. This work is supported in part by the National Science Foundation under grant numbers ANI-9806660, ANI-9809018, INT-9809903, ANI-0073725, by the Scientific and Technical Research Council of Turkey, and by the Korea Science and Engineering Foundation. An earlier version of this paper was presented at the 2000 American Control Conference.

* Corresponding author. Tel.: +90-222–335-0580; fax: +90-222-335-3616.

E-mail addresses: altug@mmf.mm.anadolu.edu.tr (A. İftar), quet.1@osu.edu (P.-F. Quet), bataslar@anadolu.edu.tr (B. Ataşlar), ozbay.1@osu.edu (H. Özbay), shivkuma@ecse.rpi.edu (S. Kalyanaraman), tskang@konkuk.ac.kr (T. Kang).

PII: S0005-1098(01)00276-X

value, and can be written in the forward or reverse direction of travel of the control cell. Many papers in the literature deal with the problem of designing the flow controllers at bottleneck nodes, see for example Altman and Başar (1997), Altman, Başar, and Srikant (1997), Benmohamed and Meerkov (1993), Jaffe (1981), Mascolo, Cavendish, and Gerla (1996); Rohrs and Berry (1997), Zhao, Li, and Sigarto (1997) and their references. ATM networks (and a competing technology MPLS (Rosen, Viswanathan, & Callon, 2001) are deployed in the core of today's Internet and therefore do not reach out to all end-systems, i.e., its control does not operate on end-to-end, short-lived flows, but on edge-to-edge, long-lived aggregate flows. As providers consider building future overlay networks on top of the Internet (where they may control both the "switches" and "end- or edge-systems"), they could consider an explicit rate-feedback framework.

In contrast, the *end-to-end* congestion control model in the Internet (e.g., TCP) attempts to solve an optimization problem by decoupling the network problem of assigning bit-marks or losses (penalties or prices) from the source problem of utility maximization (e.g., see Kelly, Maulloo, and Tan (1998), Gibbens and Kelly (1999), Kunniyur and Srikant (2001, 2000), Low (2000), Low and Lapsley (1999), Massoulie and Roberts (1999) and references within). The various versions of TCP and drop/marking algorithms (Mathis, Mahdavi, Floyd, & Romanow, 1996; Floyd & Henderson, 1999; Brakmo & Peterson, 1995; Floyd & Jacobson, 1993; Floyd, 1994) can be captured by the optimization framework just described.

Our focus in the paper is on the explicit-rate feedback framework. A challenging aspect of flow control, as far as controller design in this framework is concerned, is the existence of time-delays in the data-flow. Since the controller is to be implemented at the bottleneck node, which regulates the data rates of the sources, a time-delay occurs between the time a command signal for a rate is issued and the actual time this rate is set (time-delay from the bottleneck node to the source node, *backward delay*). Furthermore, the effect of the new rate is seen only after a time-delay which is required for the data to reach the bottleneck (time-delay from the source node to the bottleneck node, *forward delay*). Therefore, the total delay in the process (from the control input to the regulated output) is the sum of these two delays, i.e., the *round-trip delay*. To further complicate the situation, these time-delays are usually uncertain and are time-varying. Furthermore, since there usually are more than one source affecting a bottleneck, there are multiple time-delays. There are several controller design methods for different classes of systems with time-delays, see for example Kojima, Uchida, and Shimemura (1993), Niculescu, Dion, and Dugard (1996) and Stepan (1989) and their references. The techniques developed in (Foias, Özbay, & Tannenbaum, 1996; Toker & Özbay, 1995) are used in this paper.

One of the design goals considered here is *weighted fairness*, which means allocating different percentages of the available capacity to different sources. Thus, weighted fairness may be used as a pricing tool. Another design objective is *tracking*, which is to keep the queue size close to a certain desired size. By choosing this level sufficiently larger than zero and sufficiently smaller than the buffer size, nonlinear effects may also be avoided and the outgoing flow rate may be kept close to the full capacity (thus achieving the maximum utilization of the network). However, the most important design specification is *stability robustness* with respect to uncertainties in the values of time-delays in each flow path (Blanchini, Cigno, & Tempo, 1998).

In a recent work, (Özbay, Kalyanaraman, & İftar, 1998), an $\mathscr{H}^\infty$ based flow controller was designed for an explicit rate feedback based congestion control in high speed networks. Internally robust implementation of this controller was discussed in Özbay, Kang, Kalyanaraman, and İftar (1999), where some simulations demonstrating the time-domain performance of the controller were also presented. One disadvantage of that controller, however, is that, it was obtained by equalizing the nominal time-delays in all the channels. This may result in an underutilization of the network if the difference between the time-delays of different sources is large. Another shortcoming was that, only time-invariant time-delay was considered. Therefore, there was no guarantee of robustness when the time-delays vary in time. In the present work, we alleviate both of these drawbacks. Precisely, we design a controller which is robust to uncertain time-varying multiple time-delays. A multi-variable approach is undertaken here, as opposed to the single-input single-output approach undertaken in Özbay et al. (1998, 1999). Therefore, different time-delays in different channels are dealt with appropriately. The controller forces the queue length at the bottleneck node to the desired steady-state value asymptotically and also satisfies a weighted fairness condition.

In summary, our focus in the paper is on the ATM-like explicit-rate feedback framework and not the end-to-end decoupled optimization framework. We aim to use the richer information available in the former model to effect robust control over a wider spectrum of objectives in spite of time-varying time-delays.

In Section 2 we derive the mathematical model of the system and consider robustness against time-varying multiple time-delays in different channels. Performance issues are discussed in Section 3. Robustness and performance conditions are combined in Section 4 to define a two-block optimization problem, which is solved in the same section. Time-domain performance of the controller is presented in Section 5 through a number of simulations. Robustness bounds, as well as the trade-off between robustness and time-domain performance are discussed in Section 6. Some concluding remarks are made in Section 7.

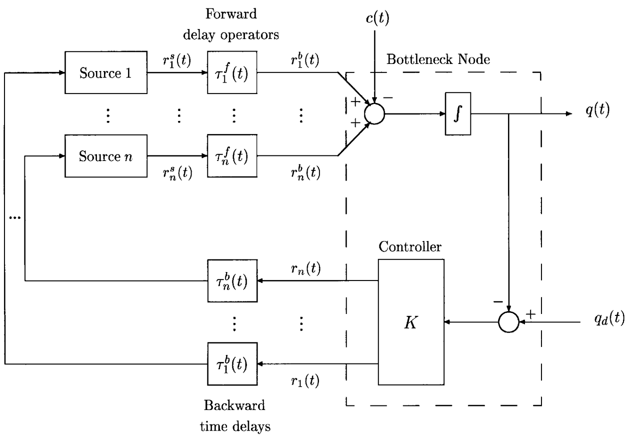

Fig. 1. The feedback control system.

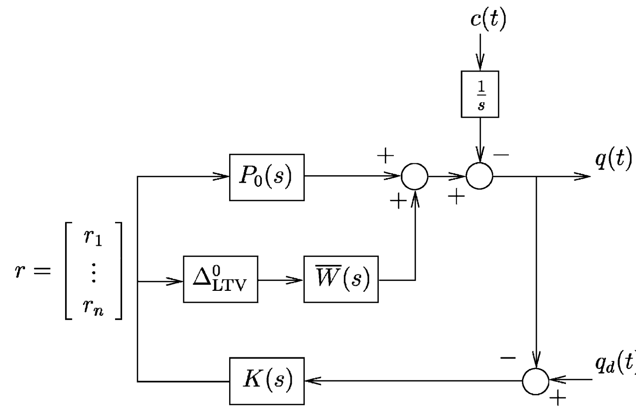

Fig. 2. Fictitious system.

2. Mathematical model

We consider the feedback system depicted in Fig. 1, which consists of a bottleneck node, n source nodes feeding the bottleneck node, and a controller which is to be implemented at the bottleneck node. A queue may form at the bottleneck node, whose dynamics is described as

$$\dot{q}(t) = \sum_{i=1}^{n} r_i^{\mathrm{b}}(t) - c(t), \tag{1}$$

where $q(t)$ is the queue length at time t, $r_i^{\mathrm{b}}(t)$ is the rate of data received at the bottleneck node at time t from the ith source node, and $c(t)$ is the outgoing flow rate from the bottleneck node at time t, which is equal to the capacity of the outgoing link at time t, unless $q(t) = 0$. In this model, the round-trip delay, $\tau_i(t)$, is defined as $\tau_i(t) = \tau_i^{\mathrm{b}}(t) + \tau_i^{\mathrm{f}}(t)$, where

- $\tau_i^{\mathrm{b}}(t) := h_i^{\mathrm{b}} + \delta_i^{\mathrm{b}}(t)$: is the backward time-delay from the controller to the ith source node (the time-delay which occurs between the time a command signal for a rate is issued and the actual time this rate is set) where h_i^{b} is the nominal time invariant known backward delay and $\delta_i^{\mathrm{b}}(t)$ is the time-varying backward time-delay uncertainty,
- $\tau_i^{\mathrm{f}}(t) := h_i^{\mathrm{f}} + \delta_i^{\mathrm{f}}(t)$: is the forward time-delay from the ith source node to the bottleneck node (the time-delay which is required for the data to reach the bottleneck node) where h_i^{f} is the nominal time invariant known forward delay and $\delta_i^{\mathrm{f}}(t)$ is the time-varying forward time-delay uncertainty.

Thus, it is assumed that the total nominal time-delay is defined as $h_i := h_i^{\mathrm{b}} + h_i^{\mathrm{f}}$ and the total time delay uncertainty is defined as $\delta_i(t) := \delta_i^{\mathrm{b}}(t) + \delta_i^{\mathrm{f}}(t)$.

Under these assumptions, to determine $r_i^{\mathrm{b}}(t)$, we write the total amount of data received at the bottleneck node from the ith source node by time t:

$$\int_0^t r_i^{\mathrm{b}}(\varphi)\,\mathrm{d}\varphi = \begin{cases} \displaystyle\int_0^{t-\tau_i^{\mathrm{f}}(t)} r_i^{\mathrm{s}}(\varphi)\,\mathrm{d}\varphi, & t - \tau_i^{\mathrm{f}}(t) \geqslant 0, \\ 0, & t - \tau_i^{\mathrm{f}}(t) < 0, \end{cases} \tag{2}$$

where

- $r_i^{\mathrm{s}}(t) := r_i(t - \tau_i^{\mathrm{b}}(t))$: is the rate of data sent from the ith source node at time t,
- $r_i(t)$: is the rate command for the ith source node issued at the controller at time t.

By taking the derivative of both sides of (2), the *forward delay operator* equations are obtained as

$$r_i^{\mathrm{b}}(t) = \begin{cases} (1 - \dot{\delta}_i^{\mathrm{f}}(t)) r_i(t - \tau_i(t)), & t - \tau_i^{\mathrm{f}}(t) \geqslant 0, \\ 0, & t - \tau_i^{\mathrm{f}}(t) < 0, \end{cases} \tag{3}$$

where we made the substitution $r_i^{\mathrm{s}}(t - \tau_i^{\mathrm{f}}(t)) = r_i(t - \tau_i(t))$ and $\dot{\tau}_i^{\mathrm{f}}(t) = \dot{\delta}_i^{\mathrm{f}}(t)$.

We assume that $(\mathrm{d}/\mathrm{d}t)(t - \tau_i^{\mathrm{f}}(t)) > 0$, then $\dot{\tau}_i^{\mathrm{f}}(t) < 1$, i.e. $\dot{\delta}_i^{\mathrm{f}}(t) < 1$. If this is not true, a packet sent out from the source node at a particular time may reach the bottleneck node before another packet which was sent at an earlier time. Furthermore, $\delta_i(t)$ and $\delta_i^{\mathrm{f}}(t)$ are assumed to satisfy

$$|\delta_i(t)| < \delta_i^+, \quad |\dot{\delta}_i(t)| < \beta_i, \quad |\dot{\delta}_i^{\mathrm{f}}(t)| < \beta_i^{\mathrm{f}} \tag{4}$$

for some known bounds $\delta_i^+ > 0$ and $0 \leqslant \beta_i^{\mathrm{f}} \leqslant \beta_i < 1$. It is also assumed that the queue size always remains between zero and the buffer capacity and $0 \leqslant r_i(t) \leqslant d_i$, where d_i is the limit at which source i can send data.

The above described system is captured by (see Appendix A for details) the uncertain system shown in Fig. 2, with

$$P_0(s) = \frac{1}{s}[\mathrm{e}^{-h_1 s} \quad \ldots \quad \mathrm{e}^{-h_n s}], \tag{5}$$

$$\Delta^0_{\mathrm{LTV}} = \mathrm{blockdiag}\left\{ \begin{bmatrix} \Delta^0_{1,1} \\ \Delta^0_{1,2} \end{bmatrix}, \ldots, \begin{bmatrix} \Delta^0_{n,1} \\ \Delta^0_{n,2} \end{bmatrix} \right\}, \tag{6}$$

$$\bar{W}(s) = [\bar{W}_1(s) \quad \ldots \quad \bar{W}_n(s)], \tag{7}$$

$$\bar{W}_i(s) = \begin{bmatrix} \dfrac{e_{i,1}}{s} & e_{i,2} \end{bmatrix}, \tag{8}$$

where $e_{i,1} := (\beta_i + \beta_i^{\mathrm{f}})/\sqrt{1-\beta_i}$, $e_{i,2} := 2\delta_i^+$, and $\Delta^0_{i,j}$ ($i = 1, \ldots, n$ and $j = 1, 2$) are arbitrary LTV systems with norm less than 1. The induced $\mathscr{L}^2$-norm of Δ^0_{LTV} is then less than $\sqrt{2}$.

To find a fixed LTI controller $K(s)$ which robustly stabilizes the system shown in Fig. 2, let us consider the following coprime factorizations of the nominal plant in $\mathscr{H}^\infty$:

$$P_0(s) = \frac{1}{s}[\mathrm{e}^{-h_1 s} \quad \ldots \quad \mathrm{e}^{-h_n s}]$$
$$= N(s)M^{-1}(s) = \tilde{M}^{-1}(s)\tilde{N}(s), \tag{9}$$

where $N(s) = \tilde{N}(s) = (1/(s+\varepsilon))[\mathrm{e}^{-h_1 s} \ \ldots \ \mathrm{e}^{-h_n s}]$, $M(s) = (s/(s+\varepsilon))I_n$, and $\tilde{M}(s) = s/(s+\varepsilon)$, where I_n denotes the $n\times n$ identity matrix and $\varepsilon > 0$ is arbitrary. Now a parameterization of all controllers $K(s)$ which stabilize $P_0(s)$ can be obtained in terms of $Q \in \mathscr{H}^\infty$ (Zhou, Doyle, & Glover, 1996).

$$K(s) = [X(s) + M(s)Q(s)][Y(s) - N(s)Q(s)]^{-1}, \tag{10}$$

where $X \in \mathscr{H}^\infty$ and $Y \in \mathscr{H}^\infty$ satisfy the Bezout identity: $\tilde{M}(s)Y(s) + \tilde{N}(s)X(s) = 1$. Since $\lim_{s\to 0} \tilde{M}(s) = 0$, to satisfy the Bezout identity we must have $\lim_{s\to 0} \tilde{N}(s)X(s) = 1$, equivalently, $\varepsilon^{-1}[1 \ \ldots \ 1]X(0) = 1$. Thus, we choose

$$X(s) = [\alpha_1 \quad \ldots \quad \alpha_n]^{\mathrm{T}}\varepsilon, \tag{11}$$

where real numbers α_i are such that $\sum_{i=1}^n \alpha_i = 1$. The entries of $X(s)$, α_i's, bring in an extra freedom in choosing the controller. Later in the paper we will show that this freedom may be used in satisfying the weighted fairness condition. Once $X(s)$ is chosen as in (11), we must choose

$$Y(s) = \tilde{M}^{-1}(s)[1 - \tilde{N}(s)X(s)]$$
$$= \frac{s+\varepsilon}{s} - \frac{\varepsilon}{s}\sum_{i=1}^n \alpha_i \mathrm{e}^{-h_i s}. \tag{12}$$

By using the small gain theorem (e.g., see Zhou et al., 1996), the closed-loop system shown in Fig. 2 is robustly stable for all $\|\Delta^0_{\mathrm{LTV}}\| < \sqrt{2}$ (hence, the actual closed-loop system is robustly stable for all time-delay variations satisfying (4)) if $K(s)$ stabilizes $P_0(s)$ (i.e., $K(s)$ is as given in (10) with $Q \in \mathscr{H}^\infty$) and $\|K(I+P_0K)^{-1}\bar{W}\|_\infty \leqslant 1/\sqrt{2}$. This condition is satisfied if

$$\|WKS\|_\infty \leqslant 1 \tag{13}$$

with $S(s) = (I + P_0(s)K(s))^{-1} = \tilde{M}(s)[Y(s) - N(s)Q(s)]$ and $W(s) = w(s)I_n$, where

$$w(s) = \sqrt{2}\left(\frac{1}{s}\sqrt{\sum_{i=1}^n e_{i,1}^2} + \sqrt{\sum_{i=1}^n e_{i,2}^2} \right).$$

3. Performance issues

In this section we will consider some performance issues related to the nominal plant.

3.1. Tracking

One of the performance objectives of rate-based congestion control is to keep the queue size, $q(t)$, as close to its desired value, $q_{\mathrm{d}}(t)$, as possible. In order to make steady-state tracking analysis, we assume that the limit $\lim_{t\to\infty} c(t) =: c_\infty$ exists, for example $c(t)$ can be a step-like function (more realistically, $c(t)$ may have infrequent sudden changes with small variations between such changes; thus, $c(t)$ may be approximated by a series of step functions). Consider $q_{\mathrm{d}}(s) = (1/s)\hat{q}(s)$ where $\hat{q}$ is an arbitrary bounded energy signal. For example if $\hat{q}$ is a pulse of finite duration, then q_{d} is a saturating ramp. Considering the nominal plant, the tracking error, $e(t) = q_{\mathrm{d}}(t) - q(t)$, satisfies the following frequency domain identity: $e(s) = S(s)(1/s)[\hat{q}(s) + c(s)]$. Since K is a stabilizing controller, and P_0 has a pole at $s = 0$, we have that $S(0) = 0$, and by the final value theorem the steady-state value of the error is

$$e_{\mathrm{ss}} = \lim_{s\to 0}([\mathrm{e}^{-h_1 s} \quad \ldots \quad \mathrm{e}^{-h_n s}]K(s))^{-1}s[\hat{q}(s) + c(s)]$$
$$= ([1 \quad \ldots \quad 1]K(0))^{-1}c_\infty$$

(note that the signal $\hat{q}(t)$ is assumed to have finite energy, so its final value is zero). Thus, at least one of the entries of the controller must have a pole at $s = 0$, in order to have zero steady-state error.

3.2. Weighted fairness

Note that the rate feedback signals are given by $[r_1(s) \ \ldots \ r_n(s)]^{\mathrm{T}} = K(s)S(s)(1/s)(\hat{q}(s) + c(s))$. It may be desired to give different steady-state weights (say $\kappa_1, \ldots, \kappa_n$) to different sources (these weights may be determined, e.g., according to a pricing policy). That is, it is desired that $\lim_{t\to\infty} r_i(t) = \kappa_i c_\infty$, where real numbers $\kappa_i > 0$ are such that $\sum_{i=1}^n \kappa_i = 1$. This means that the entries, $K_i(s)$, of the controller $K(s)$ must satisfy

$$\lim_{s\to 0} K_i(s)([\mathrm{e}^{-h_1 s} \quad \ldots \quad \mathrm{e}^{-h_n s}]K(s))^{-1} = \kappa_i \tag{14}$$

for $i = 1, \ldots, n$. If the steady-state weights must be distributed equally among the sources, we take $\kappa_i = 1/n$, $i = 1, \ldots, n$.

3.3. Transient response

Besides the steady-state behavior, it is also desired to control the transient response of the system. For this purpose the $\mathscr{H}^\infty$ norm of the weighted sensitivity function can be taken as the cost to be minimized (this corresponds to worst energy minimization for the tracking error, see for example Doyle, Francis, and Tannenbaum (1992), Foias, Özbay, and Tannenbaum (1996). More precisely, the problem is to

$$\text{minimize} \quad \|W_{\mathrm{s}}S\|_\infty \tag{15}$$

over all controllers K stabilizing P_0, where W_{s} is the sensitivity weighting filter. By examining the formula given in Toker and Özbay (1995), it is seen that the poles of this filter appear as the poles of the open-loop system P_0K. Since P_0 has one pole at $s=0$, in order for K to have a pole at $s=0$ (which was found to be a requirement for tracking) W_{s} must have double poles at $s=0$. Thus we take

$$W_{\mathrm{s}}(s)=\frac{1}{s^2}. \tag{16}$$

4. An $\mathscr{H}^\infty$ optimization problem

We can combine the robust stability (13) and nominal performance (15) conditions to define a two-block $\mathscr{H}^\infty$ optimization problem:

$$\inf\left\|\begin{bmatrix} W_{\mathrm{s}}S \\ WKS\end{bmatrix}\right\|_\infty =: \gamma_{\mathrm{opt}}, \tag{17}$$

where the infimum is taken over all K stabilizing P_0 subject to the weighted fairness condition (14). Note that, due to the choice of W_{s} in (16), the tracking condition (that K must have a pole at $s=0$) will be satisfied automatically.

The following observations will be used to determine an upper bound for γ_{opt} in terms of solutions to n decoupled $\mathscr{H}^\infty$ problems. We begin with

$$\begin{aligned} W_{\mathrm{s}}(s)S(s) &= W_{\mathrm{s}}(s)\tilde{M}(s)[Y(s)-N(s)Q(s)] \\ &= W_{\mathrm{s}}(s)\frac{s}{s+\varepsilon}\left[\frac{s+\varepsilon}{s}-\frac{\varepsilon}{s}\sum_{i=1}^{n}\alpha_i \mathrm{e}^{-h_i s}\right. \\ &\quad \left. -\frac{1}{s+\varepsilon}\sum_{i=1}^{n}\mathrm{e}^{-h_i s}Q_i(s)\right] \\ &= \sum_{i=1}^{n}\alpha_i W_{\mathrm{s}}(s)M_i(s)[Y_i(s)-N_i(s)Q_i(s)], \end{aligned} \tag{18}$$

where $Y_i(s) := (s+\varepsilon)/s - (\varepsilon/s)\mathrm{e}^{-h_i s}$, $N_i(s) := (1/\alpha_i(s+\varepsilon))\mathrm{e}^{-h_i s}$, $M_i(s) := s/(s+\varepsilon)$, and $Q_i(s)$ is the ith element of $Q(s)$. Let us also define $X_i(s) := \alpha_i\varepsilon$, which satisfies $M_i(s)Y_i(s)+N_i(s)X_i(s)=1$. Then

$$\begin{aligned} W(s)K(s)S(s) &= W(s)[X(s)+M(s)Q(s)]\tilde{M}(s) \\ &= \sum_{i=1}^{n} W_i(s)\left[\alpha_i\varepsilon+\frac{s}{s+\varepsilon}Q_i(s)\right]\frac{s}{s+\varepsilon} \\ &= \sum_{i=1}^{n} W_i(s)[X_i(s)+M_i(s)Q_i(s)]M_i(s), \end{aligned} \tag{19}$$

where $W_i(s)$ is a $n\times 1$ vector consisting of $w(s)$ at the ith row and 0's elsewhere.

Using (18) and (19), problem (17) can be rewritten as

$$\inf_{Q\in\mathscr{H}^\infty}\left\|\sum_{i=1}^{n}\begin{bmatrix}\alpha_i W_{\mathrm{s}}[Y_i-N_iQ_i]M_i \\ W_i[X_i+M_iQ_i]M_i\end{bmatrix}\right\|_\infty =: \gamma_{\mathrm{opt}}, \tag{20}$$

where the infimum is taken over all $Q\in\mathscr{H}^\infty$, subject to the weighted fairness condition (14). Let

$$\inf_{Q_i\in\mathscr{H}^\infty}\left\|\begin{bmatrix} n^{-1}W_{\mathrm{s}}[Y_i-N_iQ_i]M_i \\ (n\alpha_i)^{-1}w[X_i+M_iQ_i]M_i\end{bmatrix}\right\|_\infty =: \gamma_i, \tag{21}$$

where the infimum is taken over all $Q_i\in\mathscr{H}^\infty$, subject to the weighted fairness condition (14). Clearly, $\gamma_{\mathrm{opt}} \leqslant \sum_{i=1}^{n} n\alpha_i\gamma_i$. Since it is very complicated, if not impossible, to find an optimal solution to problem (20) subject to (14), we propose a suboptimal solution $Q(s)=[Q_1(s)\ \ldots\ Q_n(s)]^{\mathrm{T}}$, where each $Q_i(s)$ satisfies (21) such that the weighted fairness condition (14) is also satisfied. For this, we will first find a solution to (21) for each i without considering the condition (14); and then show that, by a proper choice of α_i, this solution also satisfies the weighted fairness condition (14).

Let us define

$$C_i(s) := [X_i(s)+M_i(s)Q_i(s)][Y_i(s)-N_i(s)Q_i(s)]^{-1} \tag{22}$$

so that

$$Q_i(s)=[M_i(s)+C_i(s)N_i(s)]^{-1}[C_i(s)Y_i(s)-X_i(s)]. \tag{23}$$

Then (21) can be rewritten as

$$\inf_{C_i \text{ stabilizing } P_i}\left\|\begin{bmatrix} n^{-1}W_{\mathrm{s}}[1+P_iC_i]^{-1} \\ (n\alpha_i)^{-1}wC_i[1+P_iC_i]^{-1}\end{bmatrix}\right\|_\infty =: \gamma_i, \tag{24}$$

where $P_i(s) := N_i(s)M_i^{-1}(s) = (1/\alpha_i s)\mathrm{e}^{-h_i s}$. As shown in Appendix B, the solution to problem (24) is

$$C_i(s)=\frac{n\alpha_i\gamma_i}{2\sqrt{2\sum_{j=1}^{n}(\delta_j^+)^2}}\left(\frac{sh_i-k_i}{sh_i}\right)\frac{1}{1+F_i(sh_i)}, \tag{25}$$

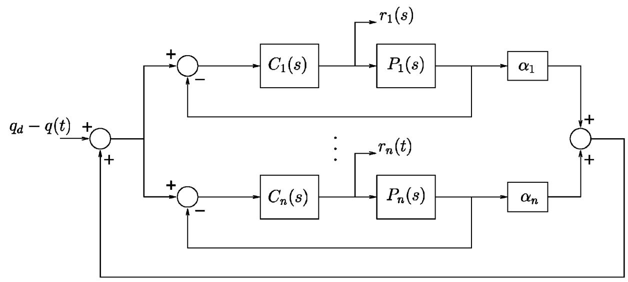

Fig. 3. The implementation of the controller.

where $F_i(sh_i)$ corresponds to a finite impulse response (FIR) filter of duration h_i (thus, it can easily be implemented in discrete-time with h_i/T_s states, where T_s is the sampling period) and k_i and γ_i are constants to be calculated. The details of the computation of F_i, k_i, and γ_i are given in Appendix B.

The controller can now be determined as (see Appendix C) $K(s) = [K_1(s)\ \dots\ K_n(s)]^{\mathrm{T}}$, where

$$K_i(s) = \frac{C_i(s)}{1 + C_i(s)P_i(s)} \left(1 - \sum_{j=1}^{n} \alpha_j \frac{P_j(s)C_j(s)}{1 + P_j(s)C_j(s)}\right)^{-1}. \tag{26}$$

The determined controller may be implemented as shown in Fig. 3.

Now it remains to be shown that the parameters α_i used in $X(s)$ can be chosen so that the resulting controller satisfies the weighted fairness condition (14). Using (10), it can be shown that (14) can be written as

$$\lim_{s\to 0} \frac{\alpha_i \varepsilon + (s/(s+\varepsilon))Q_i(s)}{\sum_{l=1}^{n} \mathrm{e}^{-h_l s}[\alpha_l \varepsilon + (s/(s+\varepsilon))Q_l(s)]} = \kappa_i. \tag{27}$$

However, since $Q_i \in \mathscr{H}^\infty$ (thus $\lim_{s\to 0} Q_i(s)$ is finite) for all $i=1,\dots,n$ and $\sum_{l=1}^{n} \alpha_l = 1$, the left-hand side simply reduces to α_i. Thus, to satisfy the weighted fairness condition, we simply need to choose: $\alpha_i = \kappa_i,\ i = 1,\dots,n$.

5. Simulation results

The closed-loop system with the determined controller shown in Fig. 3 is implemented in SIMULINK and the system is simulated for a number of different conditions.

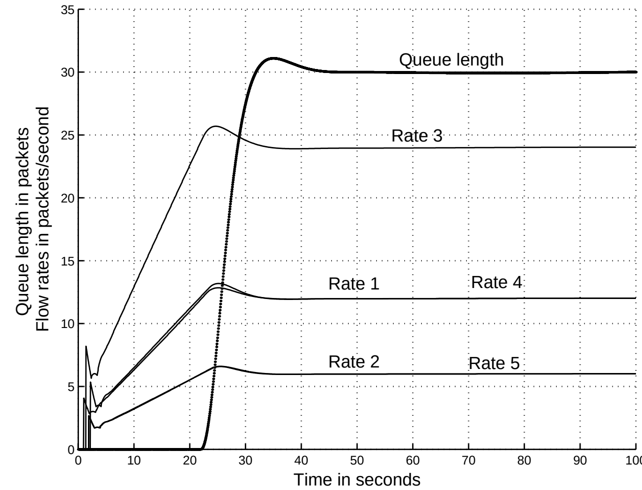

Fig. 4. Simulation results for Case 1.

Nonlinear aspects of the system are also taken into account in these simulations: the queue length and all the rates must be non-negative, and the queue size cannot exceed 100 packets. In all the cases except for Case 2 we assumed that the sources can supply data at a rate no more than 100 packets/s. The parameters $h_i = h_i^{\mathrm{b}} + h_i^{\mathrm{f}}$, κ_i, δ_i^+, β_i and β_i^{f} are design parameters used for the controller derivation and γ_i is the resulting $\mathscr{H}^\infty$ cost which is used in the controller implementation. The actual delays used in the simulations are $\tau_i^{\mathrm{b}}(t) = h_i^{\mathrm{b}} + \delta_i^{\mathrm{b}}(t)$ and $\tau_i^{\mathrm{f}}(t) = h_i^{\mathrm{f}} + \delta_i^{\mathrm{f}}(t)$. In all the cases the number of sources is assumed to be $n = 5$, the desired queue length is taken as $q_{\mathrm{d}} = 30$ packets, and the capacity of the outgoing link is taken as 60 packets/s.

Case 1: The delays (in seconds), fairness weights, and the other controller parameters are as shown in Table 1. The plots of the queue length, $q(t)$, and the flow rates, $r_i^{\mathrm{s}}(t)$, for each source are shown in Fig. 4. In the period between 0 to about 20 s we note that the queue size is zero. This corresponds to the time needed for the sum of the rates $r_i^{\mathrm{b}}(t)$ to exceed the capacity of the outgoing link at the bottleneck node. Note that the relative steady-state flow rates of the sources are equal to the relative fairness weights of these sources.

Case 2: Next we consider the same system as in Case 1, with the same delays, fairness weights, and other controller parameters. But in this case we assume that the sources can supply data at relatively lower rates. These rates are shown in

Table 1
Parameter for Case 1

i	h_i^{b}	δ_i^{b}	h_i^{f}	δ_i^{f}	κ_i	δ_i^+	β_i	β_i^{f}	γ_i
1	0.9	$0.5\sin((2\pi/50)t)$	0.1	$0.1\sin((\pi/50)t)$	0.2	2	0.1	0.01	2.35
2	1.85	$0.2\sin((\pi/50)t)$	0.15	$0.1\cos((\pi/50)t)$	0.1	3	0.2	0.02	3.07
3	0.9	$0.5\cos((2\pi/50)t)$	0.1	$0.05\sin((\pi/100)t)$	0.4	2	0.1	0.01	2.35
4	1.88	$0.3\cos((2\pi/50)t)$	0.12	$0.05\cos((\pi/100)t)$	0.2	3	0.2	0.02	3.07
5	1.8	$0.4\sin((2\pi/50)t)$	0.2	$0.05\sin((\pi/50)t)$	0.1	3	0.2	0.02	3.07

Table 2
Rate limits for Case 2

i	1	2	3	4	5
d_i (in packets/s)	10	20	20	20	5

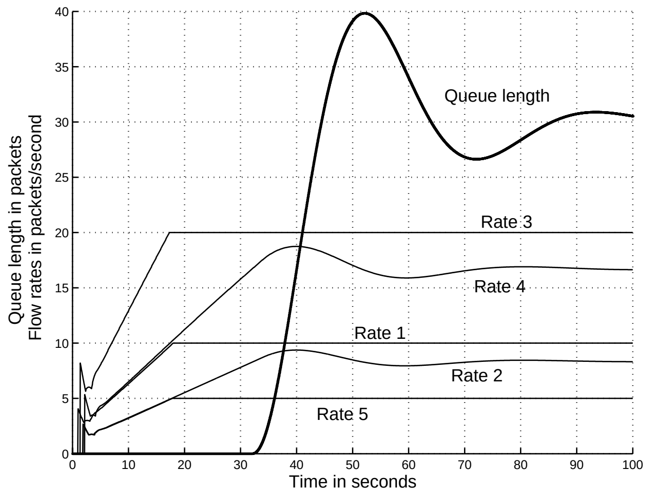

Fig. 5. Simulation results for Case 2.

Table 2 for each source. The resulting queue length and flow rates are shown in Fig. 5. As observed from this figure, flow rates at sources 1, 3 and 5 are saturated. However, the controller can successfully redistribute the unused rates to the other two sources 2 and 4. Also note that the relative steady-state flow rates of the unsaturated sources are equal to the relative fairness weights of these sources. Although the controller is unaware of the saturation, it can still regulate the queue length, however, this regulation takes a little bit more time and a larger overshoot is observed in the queue length.

Cases 3 and 4: The delays are the same as in Case 1, the problem data is changed as shown in Table 3.

The results for Cases 3 and 4 are shown in Figs. 6 and 7, respectively. Comments on these results are made in Section 6.

Although not easily noticable in the figures, steady-state oscillations exist in all the responses, due to the time-varying forward delay (precisely due to the term $(1 - \dot{\delta}_i^{\mathrm{f}}(t))$ appearing in (3)). The magnitude of these oscillations would be larger if the rate of change of the forward delay was larger and would be zero if this rate was zero (see Ataşlar et al., 2000). These oscillations are unavoidable unless some information about the forward delay uncertainty is available to the controller.

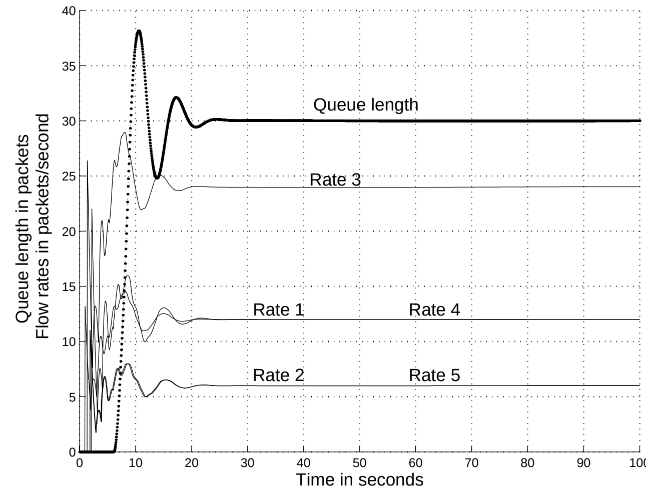

Fig. 6. Simulation results for Case 3.

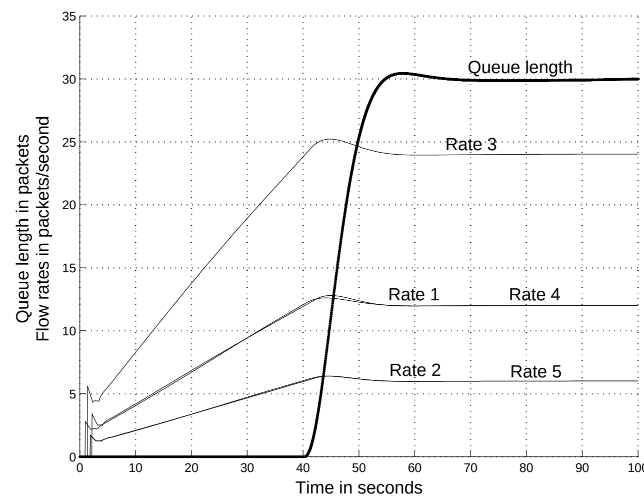

Fig. 7. Simulation results for Case 4.

Table 3
Design parameter for Cases 3 and 4

	Case 3 parameters				Case 4 parameters			
i	δ_i^+	β_i	β_i^{f}	γ_i	δ_i^+	β_i	β_i^{f}	γ_i
1	0.1	0.03	0.005	0.61	6	0.7	0.2	4.39
2	0.25	0.03	0.005	1.02	8	0.7	0.2	5.33
3	0.1	0.03	0.005	0.61	6	0.7	0.2	4.39
4	0.25	0.03	0.005	1.02	8	0.7	0.2	5.33
5	0.25	0.03	0.005	1.02	8	0.7	0.2	5.33

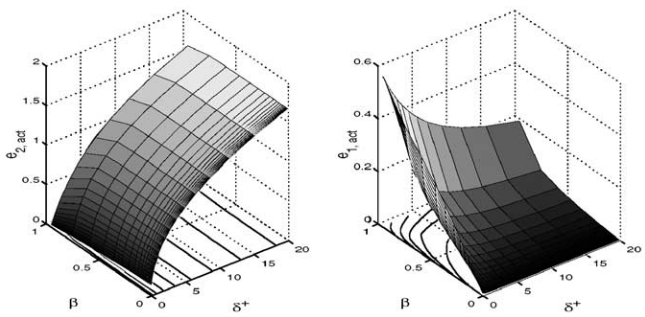

Fig. 8. Stability margins with $\beta^{\mathrm{f}} = 0$.

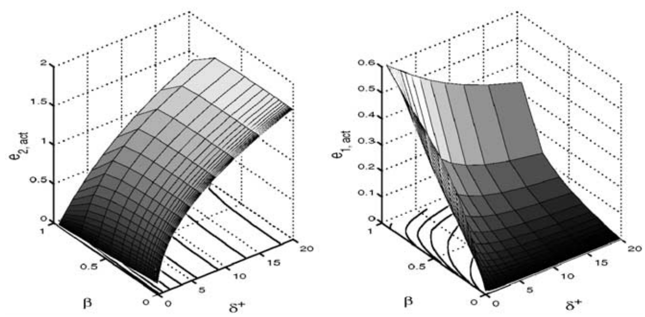

Fig. 9. Stability margins with $\beta^{\mathrm{f}} = \beta$.

We also did other simulations showing that the controller responds well to variations in the capacity of the outgoing link (see Ataşlar et al., 2000). The response of the controller in this case may further be improved if a *capacity predictor*, which predicts future values of this capacity, is included in the controller (see Quet, Ramakrishnan, Özbay, & Kalyanaraman, 2001).

6. Stability margins

Robust stability is achieved when (13) is satisfied. By using the arguments leading to (21), lower bounds (sufficient conditions) on the actual stability margins for $e_{i,1}$ (call it $e_{i,1,\mathrm{act}}$) and for $e_{i,2}$ (call it $e_{i,2,\mathrm{act}}$) must satisfy:

$$\sum_{i=1}^{n} e_{i,1,\mathrm{act}}^2 = \frac{1}{\gamma^2}\sum_{i=1}^{n} e_{i,1}^2 \quad \text{and} \qquad (28)$$
$$\sum_{i=1}^{n} e_{i,2,\mathrm{act}}^2 = \frac{1}{\gamma^2}\sum_{i=1}^{n} e_{i,2}^2,$$

where $\gamma := \sum_{i=1}^{n} n\alpha_i\gamma_i$. We note that, for the case $n > 1$, there are infinitely many solutions to (28) and the system is robustly stable for any one of these solutions. These bounds on the actual stability margins are depicted in Figs. 8 and 9 for various feasible uncertainty levels for the case $n = 1$ and where we used $h = 1$ to normalize the delay. It is seen that, for a fixed β and β^{f}, $e_{2,\mathrm{act}}$ increases with increasing δ^+ while $e_{1,\mathrm{act}}$ decreases slightly. Similarly, for a fixed δ^+ and β^{f}, $e_{1,\mathrm{act}}$ increases with increasing β while $e_{2,\mathrm{act}}$ decreases slightly. The effect of larger β^{f}, although less noticable, is to slightly decrease both margins especially when β is close to 1. These figures indicate that, for large stability robustness margins $e_{1,\mathrm{act}}$ and $e_{2,\mathrm{act}}$, the uncertainty levels δ^+ and β must be chosen as large as possible. Such a choice, however, may adversely affect the time-domain performance of the controller. For small values of β and δ^+ as for the Case 3 of simulations, it is seen in Fig. 6 that the response is oscillatory, but relatively fast. Whereas for large values of the same design parameters as for the Case 4 of simulations, it is seen in Fig. 7 that the response is smoother, but takes much longer time to settle down. Similar results were also found in Özbay et al. (1999) for the case of time-invariant delays.

7. Conclusions

Robust controller design for a flow control problem in communication networks has been considered. A robust controller has been designed against uncertain time-varying multiple time-delays. The controller brings the queue length at the bottleneck node to the desired steady-state value asymptotically and also satisfies a weighted fairness condition. Stable implementation of the controller has also been presented. This implementation is depicted in Fig. 3. As seen from this figure, the controller includes n integrators followed by delay elements (P_i's) and n blocks (C_i's) each of which include (as seen in (25)) a proportional plus integral (PI) term which is cascaded with a feedback block containing an FIR filter. Since digital implementation of delays and FIR filters are relatively easy, the controller can easily be implemented without too much computational overhead, at the bottleneck node.

A number of simulations have been included to demonstrate the time-domain performance of the controller. Stability margins for uncertainty in the time-delays and for the rate of change of the time-delays have also been discussed and their lower bounds have been derived. It has been shown that there is a tradeoff between robustness and the time-domain performance. If the uncertainty levels are chosen high, then the system is highly robust to uncertain time-varying time-delays and a smooth, however very slow, response is obtained. On the other hand, if these levels are chosen low, the response is much faster, but more oscillatory. Furthermore, for lower uncertainty levels, the actual stability margins are also lower, in general.

In this work we considered only the case of a single bottlenek node. Although the present work may be extended to the case of multiple bottlenecks, the extension is not trivial since the controllers to be implemented at different bottlenecks will interact. Initial results on this topic may be found in Biberović, İftar, and Özbay (2001).

We also note that, in this work, we did not utilize the special structure of the uncertainty given in (6). This special structure may be taken into account by using the structured singular value approach, as demonstrated in Ataşlar, Özbay, and İftar (2001).

Appendix A. Uncertainty model

From (1) and (3), $q(t)$ is given by

$$q(t)=\int_0^t\left[\sum_{i=1}^n(1-\dot{\delta}_i^{\mathrm{f}}(v))r_i(v-\tau_i(v))-c(v)\right]\mathrm{d}v+q(0). \tag{A.1}$$

Let

$$q_0(t):=\int_0^t\left[\sum_{i=1}^n r_i(v-h_i)-c(v)\right]\mathrm{d}v+q(0) \tag{A.2}$$

and $\delta_q(t):=q(t)-q_0(t)$. Also let $\lambda_i:=v-\tau_i(v)=v-h_i-\delta_i(v)=:f_i(v)$. Then

$$\frac{\mathrm{d}\lambda_i}{\mathrm{d}v}=1-\frac{\mathrm{d}\delta_i}{\mathrm{d}v}=1-g_i(\lambda_i), \tag{A.3}$$

where

$$g_i(\lambda):=\left.\frac{\mathrm{d}\delta_i}{\mathrm{d}v}\right|_{v=f_i^{-1}(\lambda)}.$$

Note that, the inverse function $v=f_i^{-1}(\lambda)$ exists since, by (4), $\mathrm{d}\lambda_i/\mathrm{d}v>0$.

Now, noting that, from (A.3), $\mathrm{d}v=\mathrm{d}\lambda_i/(1-g_i(\lambda_i))$ and assuming that $\delta_i(0)=0$, we see that:

$$\begin{aligned}\delta_q(t)&=\sum_{i=1}^n\left[\int_0^t(1-\dot{\delta}_i^{\mathrm{f}}(v))r_i(v-\tau_i(v))\,\mathrm{d}v\right.\\&\qquad\left.-\int_{-h_i}^{t-h_i}r_i(\sigma)\,\mathrm{d}\sigma\right]\\&=\sum_{i=1}^n\left[\int_0^t(1-\dot{\delta}_i^{\mathrm{f}}(v))r_i(v-\tau_i(v))\,\mathrm{d}v\right.\\&\qquad-\int_{t-h_i-\delta_i(t)}^{t-h_i}r_i(v)\,\mathrm{d}v\\&\qquad\left.-\int_0^t r_i(v-\tau_i(v))\,[1-g_i(v-\tau_i(v))]\,\mathrm{d}v\right]\\&=\sum_{i=1}^n\left[\int_0^t[g_i(v-\tau_i(v))-\dot{\delta}_i^{\mathrm{f}}(v)]r_i(v-\tau_i(v))\,\mathrm{d}v\right.\\&\qquad\left.-\int_{t-h_i-\delta_i(t)}^{t-h_i}r_i(v)\,\mathrm{d}v\right].\end{aligned} \tag{A.4}$$

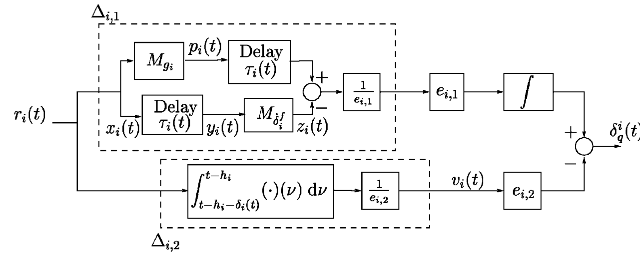

Fig. 10. Model of the uncertain part of the system.

We now have $\delta_q(t)=\sum_{i=1}^n\delta_q^i(t)$, where $\delta_q^i(t)$ is the output of the system shown in Fig. 10, with $\Delta_{i,1}$ and $\Delta_{i,2}$ linear time varying (LTV) systems, M_{g_i} and $M_{\dot{\delta}_i^{\mathrm{f}}}$ are the LTV systems defined by $p_i(t)=g_i(t)\,r_i(t)$ and $z_i(t)=\dot{\delta}_i^{\mathrm{f}}(t)\,y_i(t)$, respectively, and $e_{i,j}$'s are constants to be specified later.

Note that

$$\begin{aligned}\int_0^\infty|y_i(t)|^2\,\mathrm{d}t&=\int_0^\infty|x_i(t-\tau_i(t))|^2\,\mathrm{d}t\\&=\int_{-\tau_i(t)}^\infty|x_i(\lambda_i)|^2\frac{\mathrm{d}\lambda_i}{1-g_i(\lambda_i)}\\&<\frac{1}{1-\beta_i}\int_0^\infty|x_i(\lambda_i)|^2\,\mathrm{d}\lambda_i,\end{aligned} \tag{A.5}$$

where we assumed $r_i(t)=0$ for $t<0$. This implies that the $\mathscr{L}^2$-induced norm of $\Delta_{i,1}$ is less than $((\beta_i+\beta_i^{\mathrm{f}})/\sqrt{1-\beta_i})(1/e_{i,1})$, since $|g_i|<\beta_i$ and $|\dot{\delta}_i^{\mathrm{f}}|<\beta_i^{\mathrm{f}}$. Then, defining $e_{i,1}=(\beta_i+\beta_i^{\mathrm{f}})/\sqrt{1-\beta_i}$, we show that the $\mathscr{L}^2$-induced norm of $\Delta_{i,1}$ is less than 1.

Also, using the fact that $r_i(t)\geqslant 0$, we note that

$$\left\|\int_{t-h_i-\delta_i(t)}^{t-h_i}r_i(v)\,\mathrm{d}v\right\|_2<\left\|\int_{t-h_i-\delta_i^+}^{t-h_i+\delta_i^+}r_i(v)\,\mathrm{d}v\right\|_2 \tag{A.6}$$

since $|\delta_i(t)|<\delta_i^+$. Therefore we obtain

$$\|v_i\|_2<\left\|\frac{1}{e_{i,2}}\left(\frac{1-\mathrm{e}^{-2\delta_i^+s}}{s}\mathrm{e}^{-(h-\delta_i^+)s}\right)\right\|_\infty\|r_i\|_2. \tag{A.7}$$

Now, noting that

$$\left\|\frac{1}{2\delta_i^+}\left(\frac{1-\mathrm{e}^{-2\delta_i^+s}}{s}\mathrm{e}^{-(h-\delta_i^+)s}\right)\right\|_\infty<1$$

by taking $e_{i,2}=2\delta_i^+$ we have the $\mathscr{L}^2$-induced norm of $\Delta_{i,2}$ less than 1.

Appendix B. Solution to the optimization problem

Let us define $\tilde{W}_i(s) := (1/n)W_s(s) = (h_i^2/n)/(h_i s)^2$ and $\hat{W}_i(s) := (s/n)w(s) = \tilde{\beta}_i + \tilde{\delta}_i(h_i s)$, where

$$\tilde{\beta}_i = \sqrt{2}n^{-1}\sqrt{\sum_{i=1}^{n} e_{i,1}^2} = \sqrt{2}n^{-1}\sqrt{\sum_{i=1}^{n} \frac{(\beta_i + \beta_i^{\mathrm{f}})^2}{1-\beta_i}},$$

$$\tilde{\delta}_i = \sqrt{2}(nh_i)^{-1}\sqrt{\sum_{i=1}^{n} e_{i,2}^2} = 2\sqrt{2}(nh_i)^{-1}\sqrt{\sum_{i=1}^{n} (\delta_i^+)^2}.$$

We also define $\hat{P}_i(s) := (1/nh_i)P_i(s) = (1/n\alpha_i)(1/h_i s)\mathrm{e}^{-h_i s}$ and $\hat{C}_i(s) := nh_i C_i(s)$, so that (24) can be rewritten as

$$\inf_{\hat{C}_i \text{ stabilizing } \hat{P}_i} \left\| \begin{bmatrix} \tilde{W}_i[1+\hat{P}_i\hat{C}_i]^{-1} \\ \hat{W}_i\hat{P}_i\hat{C}_i[1+\hat{P}_i\hat{C}_i]^{-1} \end{bmatrix} \right\|_\infty =: \gamma_i, \qquad \text{(B.1)}$$

which is now in terms of the normalized frequency $\hat{s}_i := h_i s$. In deriving (B.1) from (24) we also made use of the fact that $|\mathrm{e}^{-\mathrm{j}h_i\omega}| = 1$ for all $\omega \in \mathbb{R}$.

By defining $\hat{\beta}_i = n\tilde{\beta}_i/h_i^2$, $\hat{\delta}_i = n\tilde{\delta}_i/h_i^2$, and $\hat{\gamma}_i = n\gamma_i/h_i^2$, and applying the formulae given in Toker and Özbay (1995) and Özbay et al. (1999), the optimal solution to (B.1) is found as

$$\hat{C}_i(s) = \frac{n\alpha_i\hat{\gamma}_i}{\hat{\delta}_i}\left(\frac{\hat{s}_i - k_i}{\hat{s}_i}\right)\frac{1}{1+F_i(\hat{s}_i)}, \qquad \text{(B.2)}$$

where

$$F_i(\zeta) = \frac{(\zeta+k_i)(\zeta+a_i)(\zeta^2+b_i\zeta+c_i) - (\zeta^4 - \hat{\gamma}_i^{-2})}{\zeta^4 - \hat{\gamma}_i^{-2}} - \frac{(\hat{\gamma}_i/\hat{\delta}_i)\mathrm{e}^{-\zeta}\zeta^2(\zeta - k_i)}{\zeta^4 - \hat{\gamma}_i^{-2}}, \qquad \text{(B.3)}$$

$$c_i := \sqrt{x_i}, \quad a_i := \frac{1}{c_i\hat{\delta}_i}\sqrt{1 - \frac{\hat{\beta}_i^2}{\hat{\gamma}_i^2}},$$

$$b_i := \sqrt{\frac{\hat{\beta}_i^2}{\hat{\delta}_i^2} + 2c_i - a_i^2}, \quad k_i := \frac{\rho_i - 1}{\sqrt{\hat{\gamma}_i}(\rho_i + 1)}$$

and $x_i > 0$ is the unique positive root of

$$x_i^3 + \frac{1}{\hat{\gamma}_i^2}x_i^2 + \left(1 - \frac{\hat{\beta}_i^2}{\hat{\gamma}_i^2}\right)\frac{\hat{\beta}_i^2}{\hat{\delta}_i^4}x_i - \frac{(1-\hat{\beta}_i^2/\hat{\gamma}_i^2)^2}{\hat{\delta}_i^4} = 0$$

with

$$\rho_i := \frac{\mathrm{e}^{-1/\sqrt{\hat{\gamma}_i}}}{\hat{\delta}_i(1/\sqrt{\hat{\gamma}_i} + a_i)((1/\hat{\gamma}_i) + c_i + b_i/\sqrt{\hat{\gamma}_i})}.$$

The optimal $\mathscr{H}^\infty$ performance cost $\hat{\gamma}_i$ is determined as the largest root of the equation

$$1 - \frac{\hat{\gamma}_i}{\hat{\delta}_i}\mathrm{e}^{-s}s^2\frac{(s-k_i)}{(s+k_i)(s+a_i)(s^2+b_is+c_i)}\bigg|_{s=\mathrm{j}/\sqrt{\hat{\gamma}_i}} = 0. \qquad \text{(B.4)}$$

A time domain realization of $F_i(\hat{s}_i)$ given in (B.3) can be obtained by noting that

$$F_i(\zeta) = E_i(\mathrm{e}^{-\zeta}I - \mathrm{e}^{-A_i})(\zeta I - A_i)^{-1}B_i, \qquad \text{(B.5)}$$

where

$$A_i = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \hat{\gamma}_i^{-2} & 0 & 0 & 0 \end{bmatrix}, \quad B_i = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix},$$

$$E_i = \frac{\hat{\gamma}_i}{\hat{\delta}_i}[0 \quad 0 \quad k_i \quad -1].$$

This in turn shows that the impulse response of $F_i(\zeta)$ is restricted to the time interval $[0,1]$, and hence, $F_i(h_i s)$ may be realized as a finite impulse response (FIR) filter of duration h_i, with impulse response

$$f_i(t) = \begin{cases} -(1/h_i)E_i\mathrm{e}^{(1/h_i)A_i(t-h_i)}B_i & \text{for } 0 \leqslant t < h_i, \\ 0 & \text{otherwise.} \end{cases}$$

Solution to (24) can now be obtained as $C_i(s) = (1/nh_i)\hat{C}_i(s)$, which leads to (25).

Appendix C. Stable implementation of the controller

Once $C_i(s)$ is obtained as given in (25), $Q_i(s)$ can be obtained from (23). However, in order to avoid an unstable realization of $Y_i(s)$, $Q_i(s)$ must be implemented after making some factorizations in (23). Note that $Y_i(s) = (1 - N_i(s)X_i(s))M_i^{-1}(s)$. Thus, using (23),

$$\begin{aligned} Q_i(s) &= [M_i(s) + C_i(s)N_i(s)]^{-1} \\ &\quad [C_i(s)(1 - N_i(s)X_i(s)) - X_i(s)M_i(s)]M_i^{-1}(s) \\ &= \left[\frac{C_i(s)}{1+C_i(s)P_i(s)}M_i^{-1}(s) - X_i(s)\right]M_i^{-1}(s). \end{aligned} \qquad \text{(C.1)}$$

Once $Q_i(s)$, thus $Q(s)$, is obtained, the controller $K(s)$ can be obtained by using (10). We can avoid the unstable realization of the $Y(s)$ in a similar way. By substituting (12) into (10), and using the equalities $M(s)Q(s) = Q(s)\tilde{M}(s)$, $N(s)M(s) = \tilde{M}(s)\tilde{N}(s)$ and $N(s) = \tilde{N}(s)$, we can show that $K(s)$ is obtained as

$$K(s) = Z(s)[1 - N(s)Z(s)]^{-1}\tilde{M}(s), \qquad \text{(C.2)}$$

where

$$Z(s)=\begin{bmatrix}\dfrac{C_1(s)}{1+C_1(s)P_1(s)}\\ \vdots \\ \dfrac{C_n(s)}{1+C_n(s)P_n(s)}\end{bmatrix}\tilde{M}^{-1}(s). \tag{C.3}$$

Then, using (C.3) in (C.2), the controller is determined as $K(s)=[K_1(s)\ \ldots\ K_n(s)]^{\mathrm{T}}$, where $K_i(s)$ is as given in (26).

References

Altman, E., Başar, T. (1997). Multi-user rate-based flow control: Distributed game-theoretic algorithms. In *Proceedings of 36th Conference on Decision and Control* (pp. 2916–2921). San Diego, CA.

Altman, E., Başar, T., & Srikant, R. (1997). Multi-user rate-based flow control with action delays: a team-theoretic approach. In *Proceedings of 36th Conference on Decision and Control* (pp. 2387–2392). San Diego, CA.

Ataşlar, B., Özbay, H., & İftar, A. (2001). Comparison of $\mathscr{H}^\infty$ and μ-synthesis based flow controllers for high-speed networks with multiple time-delays. In *Proceedings of the American Control Conference* (pp. 3787–3788). Arlington, VA.

Ataşlar, B., Quet, P.-F., İftar, A., Özbay, H., Kang, T., & Kalyanaraman, S. (2000). Robust rate-based flow controllers for high-speed networks: The case of uncertain time-varying multiple time-delays. In *Proceedings of the American Control Conference*, Vol. 4 (pp. 2804 –2808).

ATM Forum Traffic Management, AF-TM-0056.000. The ATM Forum Traffic Management Spec. Vers. 4.0, April 1996.

Benmohamed, L., & Meerkov, S.M. (1993). Feedback control of congestion in packet switching networks: The case of a single congested node. In *IEEE/ACM Transactions on Networking*, *1*, 693–707.

Biberović, E., İftar, A., & Özbay, H. (2001). A solution to the robust flow control problem for networks with multiple bottlenecks. In *Proceedings of IEEE Conference on Decision and Control* (pp. 2303–2308). Orlando, FL.

Blanchini, F., Lo Cigno, R., & Tempo, R. (1998). Control of ATM networks: Fragility and robustness issues. In *Proceedings of the American Control Conference* (pp. 2847–2851). Philadelphia, PA.

Bonomi, F., & Fendick, K. W. (1995). The rate-based flow control framework for the available bit rate ATM service. *IEEE Network*, *9*(2), 25–39.

Brakmo, L. S., & Peterson, L. L. (1995). TCP vegas: End-to-end congestion avoidance on a global internet. *IEEE Journal on Selected Areas in Communication (JSAC)*, *13*(8), 1465–1480.

Doyle, J., Francis, B., & Tannenbaum, A. (1992). *Feedback control theory*. New York, NY: McMillan.

Floyd, S. (1994). TCP and explicit congestion notification. *ACM Computer Communication Review*, *24*(5), 10–23.

Floyd, S., Handley, M., Padhye, J., & Widmer, J. (2000). Equation-based congestion control for unicast applications. In *Proceedings of ACM SIGCOMM* (pp. 43–56).

Floyd, S., & Henderson, T. (1999). The NewReno modification to TCP's fast recovery algorithm. Internet RFC 2582, Experimental.

Floyd, S., & Jacobson, V. (1993). Random early detection gateways for congestion avoidance. *IEEE/ACM Transactions on Networking*, *1*(4), 397–413.

Foias, C., Özbay, H., & Tannenbaum, A. (1996). *Robust control of infinite dimensional systems: Frequency domain methods LNCIS*, No. 209. Berlin: Springer.

Gibbens, R. J., & Kelly, F. P. (1999). Ressource pricing and the evolution of congestion control. *Automatica*, *35*, 1969–1985.

Jacobson, V. (1988). Congestion avoidance and control. In *Proceedings of ACM/SIGCOMM* (pp. 314–329).

Jaffe, J. (1981). Bottleneck flow control. *IEEE Transactions on Communications*, *29*, 954–962.

Jain, R. (1996). Congestion control and traffic management in ATM networks: Recent advances and a survey. *Computer Networks and ISDN Systems*, *28*, 1723–1738.

Kelly, F., Maulloo, A., & Tan, D. (1998). Rate control in communication networks: Shadow prices, proportional fairness and stability. *Journal of the Operational Research Society*, *49*, 237–252.

Kojima, A., Uchida, K., & Shimemura, E. (1993). Robust stabilization of uncertain time delay systems via combined internal-external approach. *IEEE Transactions on Automatic Control*, *38*, 373–378.

Kunniyur, S., & Srikant, R. (2000). End-to-end congestion control schemes: Utility functions, random losses and ECN marks. In *Proceedings of INFOCOM 2000* (pp. 1323–1332). Tel Aviv, Israel.

Kunniyur, S., & Srikant, R. (2001). Analysis and design of an adaptive virtual queue algorithm for active queue management. In *Proceedings of SIGCOMM 2001*.

Low, S. H. (2000). A duality model of TCP and queue management algorithms. In *Proceedings of ITC Specialist Seminar on IP Traffic Measurement, Modeling and Management*, Monterey, CA.

Low, S. H., & Lapsley, D. E. (1999). Optimization flow control, I: Basic algorithm and convergence. *IEEE/ACM Transactions on Networking*, *7*(6), 861–875.

Mascolo, S., Cavendish, D., & Gerla, M. (1996). ATM rate based congestion control using a Smith predictor: An EPRCA implementation. In *Proceedings of IEEE INFOCOM '96*, Vol. 5 (pp. 569–576). San Fancisco.

Massoulie, L., & Roberts, J. (1999). Bandwidth sharing: Objectives and algorithms. In *Proceedings of INFOCOM '99* (pp. 1395–1403).

Mathis, M., Mahdavi, J., Floyd, S., & Romanow, A. (1996). TCP selective acknowledgement options. *IETF Internet RFC*, *2018*.

Niculescu, S. I., Dion, J. M., & Dugard, L. (1996). Robust stabilization for uncertain time delay systems containing saturating actuators. *IEEE Transactions on Automatic Control*, *41*, 742–746.

Özbay, H., Kalyanaraman, S., & İftar, A. (1998). On rate-based congestion control in high speed networks: Design of an $\mathscr{H}^\infty$ based flow controller for single bottleneck. In *Proceedings of the American Control Conference*, Vol. 4 (pp. 2376–2380).

Özbay, H., Kang, T., Kalyanaraman, S., & İftar, A. (1999). Performance and robustness analysis of an $\mathscr{H}^\infty$ based flow controller. In *Proceedings of the Conference on Decision and Control*, Vol. 3 (pp. 2691–2696).

Parekh, A. K., & Gallager, G. (1994). A generalized processor sharing approach to flow control in integrated services networks: The multiple node case. *IEEE/ACM Transactions on Networking*, *2*, 137–150.

Quet, P.-F., Ramakrishnan, S., Özbay, H., & Kalyanaraman, S. (2001). On the $\mathscr{H}^\infty$ controller design for congestion control with a capacity predictor. In *Proceedings of the Conference on Decision and Control* (pp. 598–603). Orlando, FL.

Ramakrishnan, K. K., & Newman, P. (1995). Integration of rate and credit schemes for ATM flow control. *IEEE Network*, *9*(2), 49–56.

Rohrs, C. E., & Berry, R. A. (1997). A linear control approach to explicit rate feedback in ATM networks. In *Proceedings of INFOCOM '97* (pp. 277–282). Kobe, Japan.

Rosen, E., Viswanathan, A., & Callon, R. (2001). Multiprotocol label switching architecture. *IETF Internet RFC*, *3031*.

Stepan, G. (1989). *Retarded dynamical systems: Stability and characteristic functions Longman Scientific & Technical*. New York: Wiley.

Toker, O., & Özbay, H. (1995). $\mathscr{H}^\infty$ optimal and suboptimal controllers for infinite dimensional SISO plants. *IEEE Transactions on Automatic Control*, *40*, 751–755.

Zhao, Y., Li, S. Q., & Sigarto, S. (1997). A linear dynamic model for design of stable explicit-rate ABR control schemes. In *Proceedings of INFOCOM '97* (pp. 283–292). Kobe, Japan.

Zhou, K., Doyle, J. C., & Glover, K. (1996). *Robust and optimal control*. Englewood Cliffs: Prentice-Hall.

Pierre-François Quet was born in 1976 in Pau, France. In 1999 he obtained the M.Sc. degree in Electrical Engineering from The Ohio State University, USA. He received the Diplôme d'ingénieur in Electrical Engineering in 2000 from the Ecole Supérieure de Chimie Physique Electronique de Lyon, France. He is currently a Ph.D. candidate at The Ohio State University.

Banu Ataşlar was born in Eskişehir, Turkey, in 1973. She received the B.S. and the M.S. degrees in electrical and electronics engineering from Anadolu University, Turkey. She is currently completing the Ph.D. degree at the same university, where she is also a research associate in the Department of Electrical and Electronics Engineering. Her current research interests are in the area of robust control, flow control, and routing.

Altuğ İftar was born in İstanbul, Turkey, on June 28, 1960. He received the B.S. degree in electrical engineering from Boğaziçi University, İstanbul, Turkey, in 1982 and the M.S. and Ph.D. degrees in electrical engineering from The Ohio State University, Columbus, Ohio, U.S.A., in 1984 and 1988, respectively. He held teaching assistantship positions at Boğaziçi University from 1980 to 1982 and graduate teaching and research associateship positions at The Ohio State University from 1983 to 1988. He was with the Department of Electrical Engineering at the University of Toronto, Canada, from 1988 to 1992. He joined the Department of Electrical and Electronics Engineering at Anadolu University, Eskişehir, Turkey, as an associate professor in 1992 and became a professor in 1997. Dr. İftar is the recipient of the 1993 Young Investigator Award (Teşvik Ödülü) presented by the Scientific and Technical Research Council of Turkey (TÜBİTAK).

Hitay Özbay received his B.S. (1985), M.Eng (1987), and Ph.D. (1989) degrees from Middle East Technical University (Ankara, Turkey), McGill University (Montreal, Canada), and University of Minnesota (Minneapolis, USA), respectively. After a brief service at the University of Rhode Island, in 1991 he joined The Ohio State University, where he is currently a Professor of Electrical Engineering. Dr. Özbay was an Associate Editor of IEEE Transactions on Automatic Control, 1997–1999, and served on the Board of Governors of the IEEE Control Systems Society in 1999. He is currently an Associate Editor of Automatica.

Shivkumar Kalyanaraman is an Assistant Professor at the Department of Electrical, Computer and Systems Engineering at Rensselaer Polytechnic Institute in Troy, NY. He received a B.Tech degree from the Indian Institute of Technology, Madras, India in July 1993, followed by M.S. and Ph.D. degrees in Computer and Information Sciences at the Ohio State University in 1994 and 1997, respectively. His research interests are in the areas of traffic management, automated network management, multicast and multimedia networking. His special interest lies in the interdisciplinary areas between traffic and network management, control theory, economics, scalable simulation technologies and video compression. He is an associate member of the ACM and IEEE.

Taesam Kang is an Assistant Professor at the Department of Aerospace Engineering of Konkuk University. He received his B.S., M.S., and Ph.D. degrees from Seoul National University, Seoul, Korea, in 1986, 1988, and 1992, respectively. He worked for Hoseo University as an Associate Professor, before he joined the faculty of the Konkuk University in 2001. His current research interests include MEMS sensors for inertial navigation systems, autopilot design, network control, and servo control.