LE-SBCC: Loss-Event Oriented Source-based Multicast Congestion Control
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Abstract:
Congestion control is an important building block of a Quality of Service (QoS) system for multicast-based multimedia services and applications on the World Wide Web. We propose an end-to-end single-rate source-based multicast congestion control scheme (LE-SBCC) for reliable or unreliable multicast transport protocols. It addresses all the pieces of the single-rate multicast congestion control problem including drop-to-zero issues, TCP friendliness and RTT estimation. The scheme design consists of a cascaded set of filters and a rate-based additive-increase multiplicative-decrease (AIMD) module. These filters together transform the multicast tree to appear like a unicast path for the purposes of congestion control. Unlike TCP, the scheme is not self-clocking but acts upon a stream of loss indications (LIs) from receivers. These LIs are filtered to get a stream of loss events (LEs) [8] (at most one per RTT per receiver). This LE stream is further filtered to extract the maximum LEs from any one receiver. Then the scheme effects at most one rate-reduction per round trip time (RTT). A range of results (simulation and experimental) is presented and compared against the mathematical model of the scheme components. Furthermore, we have successfully adapted TFR [8] to our scheme, which is important to multimedia services desiring relatively stable rates over short time scales.

Keywords:
Multicast, congestion control, multimedia, drop-to-zero, TCP friendliness, loss event

1 Introduction

Multicast is the preferred transport mechanism for bulk data transfer to multiple receivers especially in multimedia applications and services on the internet. Applications like content distribution, streaming, multi-player games, multimedia multi-user chat/telephony, distance education etc could benefit from multicast and QoS. Multicast congestion control is the first step toward a multicast QoS architecture for the Internet. This paper presents an end-to-end single-rate congestion control scheme (LE-SBCC) for reliable or unreliable multicast transport protocols. The scheme is purely source-based because it operates on a stream of loss-indications (LIs) from the receivers and does not require any other support from network elements, receivers or in the packet format of underlying multicast transport protocols. This feature distinguishes it from recent single-rate schemes like PGMCC [21], Equation-based TFMCC [12, 25], MTCP [20], Kasper et al [15] and Golstani et al[9]. These latter schemes require some form of special support at receivers (eg: acker capability, RTT/loss estimation capability), network elements (eg: scheme-specific aggregation capability, active services) or protocol packet-formats (eg: new header fields). A purely source-based scheme can be implemented by a simple upgrade of the source of a multicast transport protocol without touching the receivers as long as they generate minimal congestion feedback. The source-based nature of the scheme does not incur extra congestion-related (S,G) state in network elements and receivers, which can be directly exploited to support multi-source multicast applications (like interactive multi-player games and online military training simulations). Although at the source we maintain some per-receiver state in the worst case, efficient implementations can reduce the average case memory requirement to below 1MB for 10,000-50,000 receivers. This scheme is modular and can allow different control policies like AIMD (for data) or TFRC (for multimedia). This holds tremendous potential in the implementation of multimedia services and applications such as live media streaming, video conferencing and video-on-demand.

The LE-SBCC scheme builds upon our earlier work GSC [17] and Bhattacharya et al’s LPRF [2, 4]. It differs from these prior schemes in that it tackles the drop-to-zero problem under high degrees of multiplexing, addresses several implementation issues and has a much faster transient response. We believe that it is the first purely source-based scheme to fully address all components of single-rate source-based multicast congestion control, i.e. drop-to-zero issues, TCP friendliness, RTT estimation, robustness and scalability up to 10,000 receivers.

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LE-SBCC belongs to the class of single-rate schemes, i.e., which operate at the rate dictated by the worst path on the tree. Similar to PGMCC [21], it is aimed at small-medium scale receiver sets (up to 10,000 receivers). There exist another distinct class of congestion control schemes which are multi-rate and receiver-based. In such schemes, the sender organizes data as multiple layers, with each layer being sent to a separate multicast group address. Receivers measure congestion and determine the number of groups they join. These schemes could be further classified as end-to-end (e.g: RLM, RLC, and FLID-DL [16, 24, 5, 7, 3]) or network-feedback driven (e.g: [14, 22]).

In contrast to our work, the network-feedback driven schemes aim to achieve max-min fairness, but require complex support at all nodes in the multicast tree. The end-to-end receiver-based approach is attractive because of its scalability to very large group sizes. But recent analysis by Gopalakrishnan et al [10] suggests that the end-to-end receiver-based approaches are coarse-grained (in terms of join/leave latency and traffic on each group) and may have stability/fairness problems. Our scheme does not fall into this class, and is fine-grained in terms of control and traffic granularity. The combination of source-based and receiver-based approaches is largely an open problem.

2 LE-SBCC: Scheme Description

![Diagram of LE-SBCC Scheme]

Figure 1: CASCADED FILTER MODEL OF LE-SBCC

The LE-SBCC scheme consists of a purely source-based cascaded set of filters and RTT estimation modules feeding into a rate-based additive increase/multiplicative decrease (AIMD) module (Figure 1). These filters, elucidated in section 2.1, together transform the multicast tree to appear like a unicast path for the purposes of congestion control. Also, unlike TCP, the scheme is not self-clocking but acts upon a stream of loss indications (LIs) from receivers. The filters are then designed to address the following key problems:

**Drop-to-Zero:** is the problem of reacting to more loss indications (LIs) than necessary leading to a beat-down of the multicast flow’s rate[25, 4]. This occurs because the multicast flow receives LIs from multiple paths and may not filter LIs sufficiently. In an earlier work [17], we showed that without flow multiplexing on paths, the problem of independent multi-path loss can be addressed through a simple adaptive time filter and a TCP-like RTT/RTO estimator. A single flow’s congestion control actions directly impacts the loss rate and loss patterns in this case. However, this solution is insufficient if paths display a high degree of flow multiplexing, because then any single flow’s congestion actions do not appreciably impact the path loss rates.

**TCP-unfriendliness:** is the problem of reacting to less LIs than a hypothetical TCP flow would on the worst loss path [6, 25, 12]. TCP-friendliness is a subject of current debate [8, 21, 12]. In particular, Padhye et al’s TCP throughput formula [18] requires the definition and estimation of “loss rate” and “RTT” of the paths. The unicast scheme, TFRC [8] introduced the concept of loss events (LEs) and uses the LE rate instead of the LI rate as the “loss rate.” An LE is a binary number defined every RTT per receiver which is 1 when one or more LIs are generated in that RTT by the receiver, and is 0 otherwise. Like TFRC [8], this paper will also first derive LEs from LIs before responding to congestion.
**RTT estimation:** PGMCC and TFMCC [21, 12] have receivers estimate RTT and drive the control based upon paths having the maximum value of \( RTT \sqrt{p_t} \), where \( p_t \) is the per-packet LE probability. Our proposed scheme LE-SBCC computes \( \text{max}RTT \) and \( \text{max}p_t \), concurrently and uses both these estimates to drive the AIMD module. This decoupled use of \( \text{max}RTT \) and \( \text{max}p_t \) represents a sub-optimal approach in general[21, 12, 25]. However, our RTT-estimation approach tends to measure the \( \text{max}RTT \) from the *currently congested sub-tree*, which reduces this suboptimality in practice.

### 2.1 Cascaded Filter Model

Figure 1 illustrates the outline of the building blocks of the scheme implemented at the multicast source. The cascade of filters converts a stream of loss indications (LIs) from receivers into per-receiver LEs, filter the LEs further, and finally outputs a stream of rate-reduction indications (RRs) to the rate-based AIMD module (at most one RR per RTT). The AIMD module performs multiplicative rate decrease (MD) upon receipt of a RR and performs additive rate increase (AI) each time a RR is not received within an increase interval. Observe that this closely models the behavior of AIMD control on a *single path* where RRs are triggered by LEs [8]. The modules work as follows:

![Diagram](image)

**Figure 2: LOSS INDICATION TO LOSS EVENT FILTER (LI2LE)**

**LI2LE filter:** This filter converts per-receiver loss indications (LIs) into per-receiver loss events (LEs). Recall that an LE is a per-receiver binary number which is 1 when one or more LIs are generated per RTT per receiver, and 0 otherwise. The source stores a *per-receiver timestamp* (\( T_{LastPassed} \)) which records the time when the last LI was converted and passed as an LE. If a new LI arrives from the receiver after a period \( SRTT + 2\sigma \) of \( T_{LastPassed} \), it is converted into an LE and passed, and the timestamp \( T_{LastPassed} \) is updated to the current time. Else the LIs are filtered. This filter is critical because the rate-based AIMD module is not self-clocked (no acks like TCP, PGMCC [21] or Golsetani [9]); which leads to large burst losses with drop-tail queues (see section 3.1.3).

**Max-LPRF:** The goal of this filter is to pass, on the average, the number of LEs corresponding to what the source would have received from the worst case receiver. This worst-case loss rate estimation problem has been studied earlier in the literature (see [2] and references therein). Our filter is an extension of Bhattacharya et al’s Linear Proportional Response Filter[2] (LPRF). The LPRF is a filter which passes loss indications (LIs) with a probability \( \frac{L_i}{\sum L_i} \), where \( L_i \) is the number of LIs from receiver \( i \).

We found that we could not use the LPRF filter directly because of a number of issues. Firstly, it was originally presented under Markovian model assumptions [2] which do not hold under high multiplexing conditions (eg: see section 3). Therefore we found that it was susceptible to the drop-to-zero problem. Second, *LPRF* did not quickly adapt to sudden increases in worst-case loss rates. For example, consider a steady state case with a large number of receivers, where each receiver has a loss rate of 1%. If one of the

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\(^3SRTT = \text{average of RTT samples similar to the variable used in TCP} \)
receivers suddenly starts experiencing a loss rate of 10%, then LPRF converges to the “worst” receiver very slowly. Third, LPRF works with LIs instead of LEs. Our subsequent arguments show that using LEs is much superior to using LIs. Fourth, it does not specify an RTT estimation procedure.

The Max-LPRF works as follows: Assuming \(X_i\) is the count of LEs from receiver \(i\), this probabilistic filter takes as input all the LEs from receivers \(\sum X_i\) and on an average passes the maximum number of LEs from any one receiver (i.e. \(\max X_i\)). In particular, it passes each LE with a probability \(\frac{\max X_i}{\sum X_i}\). The LE counts per receiver \(X_i\) are decayed periodically by 10% every 100 \(SRRT\)s. The \(O(N)\) state requirements of both L1LE and Max-LPRF are not a big issue because single-rate schemes are typically targeted for a small-medium scale (< 10000 receivers). Max-LPRF tracks the worst path better than LPRF and is the crucial building block for drop-to-zero avoidance. It operates on per-receiver LE counts since they differ dramatically from LI counts in drop-tail queueing networks with no self-clocking.

**Figure 3: MAX-LPRF: MAX-LINEAR PROPORTIONAL RATE FILTER**

**Adaptive Time Filter (ATF):** This filter shown in Figure 4 is a modification of the time-filter introduced in our earlier work [17]. This filter simply drops excess LEs passed by Max-LPRF in any RTT to enforce at most one rate reduction (RR) per \(SRRT + 4\sigma\). In addition, the filter also imposes a “silence period” of \(\frac{1}{2}(SRRT + 4\sigma)\) when no packets are sent. As suggested in [17], the goal is to reduce the probability of losing any control traffic or retransmissions during this phase.

**RTT Estimation:** All filters and the AIMD module need RTT estimates which is fed by the RTT estimation module developed in our earlier work [17]. It works similar to the TCP timeout procedure i.e. it calculates a smoothed RTT (\(SRRT\)) and a mean deviation which approximates the standard deviation \(\sigma\). However the set of samples is pruned to exclude a large fraction of samples which are smaller than \(SRRT/2\) (i.e. smaller by an order of magnitude) to bias the average RTT higher. The rate increase uses intervals of length \(SRRT + 2\sigma\) (see [17]) while other functions differ in their use of \(\sigma\) as described earlier. Observe that this LI-driven RTT estimation procedure will tend to opportunistically measure the worst RTT of paths which are
generating more LIs. That is, the procedure tends to estimate the worst RTT from the currently congested sub-tree. Therefore, the de-coupling of RTT estimation from worst-case loss rate estimation does not lead to significant suboptimality in practice.

In summary, for ideal operation, the scheme expects: (a) at least one LI per receiver seeing loss per RTT if packets are lost in that RTT (i.e. at least one LE per RTT) and (b) timely generation/forwarding of feedback by receivers/network elements to allow reliable RTT sampling.\(^4\) In general, if expectations (a) and (b) are not satisfied completely or satisfied in an unreliable manner, the scheme performance will degrade. In particular, the scheme is sensitive to arbitrary delays introduced by receivers/network elements in generating an LI corresponding to a lost packet. Further LI aggregation like in PGM [23] itself is a form of filtering which suppresses receiver IDs, timing information and reduces LI or LE counts, thus conflicting with (a) and (b) above\(^5\). We examine such performance effects in the following sections. The pseudo code for the scheme is presented in Appendix A.

3 Performance Evaluation

We have evaluated the performance of LE-SBCC to test for Drop-to-Zero avoidance, TCP-friendliness and LI aggregation effects. We use the following methods to test our scheme:

1. Simulations to observe the detailed scheme dynamics and background TCP dynamics for tens of receivers, and to explore the scheme performance for up to 10000 receivers.
2. Simple Markov chain based modeling to obtain a better understanding of the choice of LEs as opposed to LIs in our scheme.
3. Linux-based implementation/experimentation (high multiplexing degrees and up to tens of receivers) to understand implementation issues and test performance on a real network.

3.1 Evaluation: Drop-to-Zero Avoidance

Recall that Drop-to-Zero Avoidance is the problem of reacting to more loss indications (LIs) than necessary leading to a beat-down of the multicast flow’s rate. Drop-to-Zero problem occurs in the following scenarios:

1. High multiplexing, where many flows share a common bottleneck; and the multicast flow receives independent feedback from several such paths. In such a case, the loss rates observed at the source are independent of sending rate, and the multicast flow receives feedback from several paths which requires filtering.
2. Large receiver sets with heterogeneous loss probabilities for different receivers result in a huge number of LIs. It becomes important that the scheme does not react to more LIs than that generated by the worst-loss receiver.
3. LI aggregation leads to degradation of loss and timing information as some LIs are suppressed by the intermediate aggregator. Aggregation effects are discussed in a later section.

We have analyzed the drop-to-zero issue using simulations and Markov Chain based theoretical analysis.

3.1.1 Simulations Illustrating Drop-to-Zero Avoidance

Consider a set of reliable multicast (RM) flows that goes to receiver 1 through receiver 4 as shown in Figure 5. The capacity of the links connecting the sources (on the left) to “Router 1” are 6 Mbps each. The link from “Router 1” to “Router 2” has 6 Mbps for each flow (totally 102Mbps). There is a Main RM source which sends data to receivers 1 through 4. The links from “Router 2” to each of the receivers are bottlenecks at 5 Mbps each.\(^4\)

\(^4\)Within these constraints the scheme may be successfully applied to unreliable multicast transport protocols.

\(^5\)LIs carry RTT information which affects the RTT estimation, and LEs carry receiver IDs which affects the max-LPRF filter.
A set of 4 background unicast flows implementing the LE-SBCC algorithm compete on each of the 4 bottlenecks links. The unicast flows in “Set 1” through “Set 4” compete on the bottleneck links to “Router 2-Receiver 1” through “Router 2-Receiver 4” respectively. The fair rate for all flows is therefore 1 Mbps. The buffer size (125 pkts) is roughly twice the bandwidth times fixed delays. The packet size is 560 bytes. This topology tests the performance of the multicast congestion control scheme under independent loss rates on paths, and reasonable degrees of multiplexing on each path. In effect, the loss rate on such congested paths is less dependent on the rate change of any single flow on the path (especially the multicast flow).

Figure 6 (rate graph) shows that the multicast flow rate (solid line) competes fairly with a selected subset of unicast background flows (dotted lines) and the average rate is close to 1 Mbps, the fair share. More importantly, the multicast flow does not get beaten down because of the independent loss indications received from each path. This is a non-trivial illustration of drop-to-zero avoidance. Observe that the same topology also illustrates the basic applicability of LE-SBCC to multi-sender multicast, because the competing unicast flows use the same congestion control scheme. Each sender’s multicast traffic is controlled as if it were the only sender.

The analysis of the number of packets transmitted, the number of LIs and LEs and rate reductions (RRs) also illustrates the impact of the cascaded set of filters. In particular, the main RM flow transmitted a total of 167762 pkts in 450s and received 949 LIs. After LI2LE filtering, there were only 32 LEs, a dramatic reduction! The Max-LPRF and ATF further filter LEs to result in a total of 12 rate-reductions. When we go back to the graph in Figure 6 and count the number of rate-reductions in multicast as well as unicast flows, we observe that all flows have 12 rate-reductions. This is another measure of the fairness of the scheme to AIMD-based background traffic. The issue of TCP fairness is further explored in section 3.2.

As discussed in the next section, the dramatic difference between the number of LIs and LEs is because of bursty losses caused by the lack of self-clocking (unlike TCP), rate-based nature of control and the use of sizable drop-tail buffers.
3.1.2 Effects of Removing a Filter From the Cascade

(a) Removing both L12LE and maxLPRF filters  
(b) Removing maxLPRF filter, and  
(c) Removing only the L12LE filter.

In all these experiments all other parameters are kept constant as before. The results are illustrated in Figure 7. Clearly, we observe the drop-to-zero problem to some degree in all cases (solid curves always below dotted curves). The results are also affected by the fact that in some cases the max-LPRF operates on LI. In these cases, the max-LPRF processes a larger absolute population of samples, and deals with a different relative distribution of counts from receivers (which affects the critical ratio: \( \frac{\max X_i}{\Sigma X_i} \)). Thus each filter in the cascade has a key role in filtering of LIs and avoiding the drop-to-zero problem.

3.1.3 Theoretical Analysis of LE vs LI Probability

Recall that our scheme uses LEs instead of LIs for the max-LPRF filter and ultimately in rate reduction decisions. Therefore, our scheme is critically dependent on the number of LEs passed by filters at the source. Thus it becomes important to study the relationship between LIs and LEs to get an understanding of when it is really important to use LEs instead of LIs.

This section presents a simple theoretical analysis to show that using LEs in general leads to a reduced probability of drop-to-zero because the number of LEs is always less than the number of LIs. Moreover, under bursty loss conditions, the number of LEs is significantly less than the number of LIs. This justifies the presence of LI to LE filter used in the cascade. Also, observe that the ratio \( \frac{\max X_i}{\Sigma X_i} \) (used in MaxLPRF) could be very different if \( X_i \) represented the count of LEs vs if it represented the count of LIs. These two reasons justify the use of LEs instead of LIs.

Consider the following Markov Chain model for unicast transmission. Assume that the RTT is constant and that the system moves in cycles of RTT. In each RTT, the source transmits with a constant rate, and may experience one or more packet losses. At the end of each RTT, it changes its rate based upon the AIMD policy and depending upon the receipt of LEs. Therefore the system may be modeled as a Markov chain with N discrete states (from 1 to N), where the state \( x \) specifies the number of packets sent during that RTT. Therefore, \( x = Rate(x) \times RTT \). The process starts in state 1, and may go up to the maximum allowable rate (which corresponds to state N). When a loss event (LE) is encountered within an RTT, the system transitions to state \( \lfloor x/2 \rfloor \). When there is no loss event in an RTT, the system moves to state \( x + 1 \). This Markov chain model is illustrated in Figure 8. The goal of this Markov chain is to estimate the ratio of per-packet LI probability \( p_{L1} \) to per-packet LE probability \( p_{L0} \), i.e., \( E(p_{L1})/E(p_{L0}) \). We analyze this chain for two cases: the non-bursty and bursty packet loss case.
Non-bursty Loss Case

We assume that the packet loss probability is uniform. Define $p$: the probability that a packet is lost, and $a$: $1-p$. In this case $p_L = p$. In state $x$, the probability of having at least one loss is $1 - a^x$. The LE probability ($p_{LE}$) is thus $\sum s(x)(1-a^x)$ where $s(x)$ is the probability of being in state $x$. Let $m = \lceil N/2 \rceil$. We have five cases to consider:

1. We are in state 1. We could increase the rate and move to 2. We could also get losses at states 2 or 3, and move to 1. We could also get a loss, and stay in 1. In steady state,

   $$s(1) \times a = s(2) \times (1 - a^2) + s(3) \times (1 - a^3)$$

2. We are in any state $x$ between 2 to $m - 1$. This is the general case mentioned above. In steady state,

   $$s(k) = s(k-1) \times a^{k-1} + s(2k) \times (1 - a^2)^k + s(2k+1)(1 - a^{2k+1})$$

3. We are in state $m$. We could increase the rate and move to $m + 1$. We could also get loss at state $N$ only, and move to $m$. In steady state,

   $$s(m) = s(m-1) \times (a^{m-1}) + s(N) \times (1 - a^N)$$

4. We are in any state $x$ between $m + 1$ to $N - 1$. The only difference between this state and 3 is that we cannot enter this state by a rate reduction, as the $N = \text{max(rate} \times \text{RTT})$. In steady state,

   $$s(k) = s(k-1) \times a^{k-1}$$

5. We are in state $N$. We cannot increase the rate further. The only way to leave this state is by cutting its rate, and moving to state $m$. In steady state,

   $$s(N) \times (1 - a^N) = s(N-1) \times a^{N-1}$$

We did a Matlab analysis to numerically solve the chain for different large $N$’s. The solutions yield the steady state probabilities $s(x)$ and hence the probability $p_{LE}$. We then plot the ratio of LI probability $p_L$ and LE probability $p_{LE}$ ratio in Figure 10 (a). The plot shows that there is not much difference between LI and LE probability. In particular, the maximum ratio of LI to LE probability is less than 1.25. In other words, the number of LEs is at most 25% smaller than the number of LIs (which happens when the LI rate is 30-40%, a large absolute loss rate).

The uniform loss probability assumption made above is not valid in bursty packet loss scenarios (though it could apply to RED gateways). The combination of our scheme with a drop-tail buffer leads to a very bursty packet loss pattern, which yields a very different picture of LI vs LE probability.

Bursty Loss Case

Our scheme uses a rate-based AIMD policy, which is not self-clocked (unlike TCP). This lack of self-clocking is because the scheme depends only upon LIs and not a stream of acks for its basic operation. A simple control-theoretic analysis [19] shows that a rate-based scheme with linear rate-increase, and lack of self-clocking leads to a bottleneck queue which grows quadratically. Therefore, if the bottleneck has a larger
buffer, the queue increases quadratically for a longer period of time. Moreover, when the buffer is full and packets are about to be dropped, the sending rate differs significantly from the optimal transmission rate. So, when the queue overflows, the number of packets dropped corresponds to \((R - C)T\) where \(R\) is the aggregate transmission rate (which is much larger than \(C\)) and \(C\) is the bottleneck capacity, and \(T\) is the round trip time (RTT) including the queuing delay. This leads to a large burst of packets dropped during the RTT before the receivers send LIs and the sources detect congestion. Therefore it is important to understand how the number of LEs differ from the number of LIs under such bursty loss conditions. We show that under such conditions, the choice of LEs is even more justified.

We use the Markov chain model as used earlier, albeit with some changes. To approximate the bursty loss behavior, we assume that if a packet is lost during any instant within the RTT, all remaining packets transmitted during that RTT are also lost. This assumption is also made by Padhye et al [18] for TCP throughput modeling. Under this assumption, the probability \(p\) assumes different semantics. \(p\) here is defined as the probability that a packet is lost given that either it is the first packet in the RTT or the preceding packet in the RTT is not lost. Assume like before that \(a = 1 - p\).

![Figure 9: p's SEMANTICS IN BURSTY LOSS CASE](image)

In this case, \(p \neq p_{li}\) (see Figure 9) because \(p_{li}\) is affected by every packet loss unlike \(p\). However, the overall real LI probability \(p_{li}\) can be derived from \(p\). First, recall that the state \(x\) specifies the number of packets transmitted during a certain RTT. Assuming that the first \(i\) packets are not lost, \(x - i\) is the number of lost packets in that RTT. Under our assumption, during a RTT,

\[
P(\text{The first } i \text{ packets are not lost and the subsequent packets in the round are lost}) = (1 - p)^i p
\]

Therefore, at state \(x\), the expected number of lost packets is \(\sum_{i=0}^{x} (x - i)(1 - p)^i p\). The LI probability \(p_{li}\) is the expected number of lost packets divided by the expected total number of packets transmitted:

\[
p_{li} = \frac{\sum_{x=0}^{\infty} s(x) \sum_{i=0}^{x} (x - i)(1 - p)^i p}{\sum_{x=0}^{\infty} x \cdot s(x)}
\]

At state \(x\), the expected number of loss events (LE) is the probability that there is at least one loss, that is \(1 - (1 - p)^x\). Therefore,

\[
p_{le} = \sum_{x=0}^{\infty} s(x) \cdot (1 - (1 - p)^x)
\]

As a result, ratio of LI and LE probabilities \(\frac{p_{li}}{p_{le}}\) is substantially altered. This is illustrated in Figure 10 (b). The figure plots the ratio as \(p_{li}\) ranges from 0 to 1. The ratio \(\frac{p_{li}}{p_{le}}\) is huge when \(p_{li}\) is small (which is the common case of network operations). This is because the small overall LI probability leads to concentrated burst loss in few RTTs leading to a small number of LEs in comparison to LIs.

For lower loss probabilities, the \(p_{li}/p_{le}\) values are higher. To see the relation between LI and LE more clearly in this region, we have plot another graph for the low LI probability region (Figure 10 (c)). We observe high \(p_{li}/p_{le}\) ratios (9-90). Comparing these numbers to the numbers of LIs and LEs observed in simulation (see section 3.1.1), we observe that the model gives the same order of magnitude of results as the simulation.
results. In that case \( p_{li}/p_{le} = 949/32 = 29.7 \). Therefore, we can conclude that the LI2LE filtering is critical in our cascade filter design.

![Diagram](image)

**Figure 11:** TOPOLOGY: LARGE HETEROGENEOUS RECEIVER SETS

### 3.1.4 Drop-to-Zero Avoidance in Medium-Large Scale Trees

Recall that large receiver sets result in increased number of LIs being generated. To test the performance of our scheme in such scenarios we gradually increase the number of receivers in the multicast session, and run simulation for each addition with the assumption of constant RTTs and that each path experiences a uniform loss probability. We use a topology shown in Figure 11 where all the receivers and the single source are connected to the same router. One of the receivers in the session has the worst loss rate, and the rest have lower loss probabilities. We expect that the number of LEs passed by source for rate reduction should remain fairly constant and equal to the number of LEs generated by the worst loss receiver.

The number of receivers in the multicast tree is increased from 1 to 10001 in a gradual manner. We assume one receiver has 10% loss, all others have 2% loss and study the effect of increasing the number of 2% receivers. The above numbers and topology were chosen as they represent a general case where one of the receivers (the 10% one) clearly has the worst loss rate. The expected number of LEs passed by the filters should be almost equal to the number of LEs generated by the worst loss receiver. The number of LEs passed for rate reduction by the source is plotted against the number of receivers (Figure 12(a)). The experiments were repeated for different random seeds and 99% confidence intervals are plotted. Under no aggregation, the number of LEs passed by source remains fairly constant (500-520). The number of LEs passed are consistent with the non-bursty theoretical analysis performed above. The expected number of LIs by the 10% receiver would be \( 0.1 \times 6000 = 600 \). By referring to the LI probability \( p_{li} \) vs LE probability \( p_{le} \) graph (Figure 10(a)) for the non-bursty model, we see that for 10% loss
The $p_s/p_c$ ratio is around 1.2. Therefore the expected number of LEs would be $600/1.2 = 500$. This number agrees very closely with the numbers obtained by simulation. We also repeated the same experiment with the max loss receiver having a loss rate of 20%. Again, the number of LEs passed (Figure 12(b)) remains fairly constant (around 1000). This indicates that under same RTT, fixed uniform loss probabilities, and no aggregation conditions our scheme passes a fairly constant number of LEs even when large number of receivers are present. This number is equal to the LEs generated by the worst loss receiver. Thus we achieve Drop-to-Zero avoidance in medium-large scale trees.

![Scalability analysis](image)

Figure 12: SCALABILITY ANALYSIS

### 3.2 Evaluation: TCP friendliness

TCP-friendliness is a subject of current debate [8, 21, 12]. A good measure of TCP-friendliness would be the average throughputs of the competing TCP and RM flows. To illustrate TCP-friendliness, we use two topologies, a) where a common bottleneck is shared by many RM flows and a single TCP flow and, b) many TCP flows and a single RM flow. LE-SBCC would be TCP-unfriendly if either of the TCP or RM flow is beaten down. In Figure 5, consider two cases: case (a) when one of the flows of “Set 4” is replaced by a TCP Reno flow while the rest are LE-SBCC flows; and case (b) when all the flows in “Set 1” through “Set 4”) are replaced by TCP flows.

**Simulation results:** The simulation results for these two cases is shown in Figures 13(a) and 13(b). The dips by the TCP flow close to zero represent timeouts. Again observe that the main RM flow (solid line in both graphs) shares the bottleneck(s) fairly with the competing TCP flow(s). This becomes clearer by observing that the rate graphs for the RM and TCP flows oscillate about roughly the same mean and with the same variance. The horizontal dotted line is the expected TCP throughput plotted post-simulation using the simplified TCP equation ($1.22MSS/RTT \sqrt{p_c}$), where $p_c$ is the LE probability of the worst-loss receiver. Recall that for the purpose of congestion control, we aim to reduce the multicast tree to a unicast path with the worst-loss receiver being the only one with LE probability $p_c$ and a worst $RTT$. Clearly from the Figures 13(a) and 13(b), the rate values oscillate about the theoretical mean shown by horizontal dotted line. This fact is a good measure of the TCP friendliness of our scheme, further demonstrates the validity of choosing LE rather than LI probabilities in our scheme.

**Experimentation results:** The scheme is implemented in a Linux 7.0 testbed. The topology used is identical to the one used for the simulations as described above. Again the RM flows are fair to the competing TCP flow (Figure 13(c)). More details on this are presented in the section on implementation issues.

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6To avoid cluttering, the TCP throughput is sampled every 2s
3.2.1 DropTail vs. RED Queues

We evaluated the TCP-friendliness performance of our scheme with the two queue management policies - Tail Drop and Random Early Detection (RED). These are the two widely used queue management policies and it is important to test the performance of LE-SBCC with both the policies. The topology used for this purpose is the same as in the above experiment with the following key parameters: TCP version: Reno, Queue size = 125 packets, Packet size = 560 bytes.

![Graphs](image.png)

**Figure 14: DROPTAIL VS. RED: RATE GRAPHES**

As we can see from Figure 14, the throughput is marginally lower with RED than with DropTail and so is the measured RTT (Figure 15). (The absolute RTT values are smaller in RED, because when losses occur, RED has lower average queue length.) Also observe that the RTT converges much faster with RED than DropTail. This is because the RED policy drops packets before the queue is full resulting in more frequent and timely loss indication from the receivers. The scheme remains fair towards the competing TCP flow in both cases and we do not observe any drop-to-zero with RM flows.

3.2.2 Different Loss Paths

It was speculated that the scheme would not perform as well with the multicast tree having paths with varying degree of loss probabilities and RTTs. We performed the following simulation to test the robustness of our scheme under these heterogeneous conditions. Again the topology remains the same as the above experiment with modifications in following key parameters: The buffer is 125 packets on all links except that it increases on the four links from the router to receiver 1, receiver 2, receiver 3 and receiver 4 with the values 25, 50, 75 and 100 respectively.

Having different buffer sizes is an indirect way to affect RTT and loss probabilities on a link. The link from router to receiver 1 has the smallest buffer (25 packets). Therefore, it has the smallest RTT and the largest loss rate. On the other hand, the link from router to receiver 4 has the largest buffer (100 packets). So it has the largest RTT and smallest loss rate. For the Main Source, MaxLPRF will respond to LEs from receiver 1, whereas
its RTT would be biased higher (average of the 4 different RTTs measures received from the 4 receivers). The simulation is performed for both non-aggregation (Figure 16) and aggregation cases (Figure 17). The graphs clearly demonstrate that the algorithm works well under heterogeneous loss probability and RTT conditions and with aggregation enabled at the router too.

### 3.3 Evaluation: LI Aggregation Effects on Performance

Aggregation of loss indications (LIs) has an effect on the number of LEs and timing information reaching the source, and therefore affects the performance of LE-SBCC. To evaluate these effects, consider the simple multi-level topology in Figure 18. As the number of receivers and their loss rates are varied, consider three cases which we simulated:

1. **No LI Aggregation** performed at the routers.
2. **Partial LI Aggregation**: LI aggregation performed at router 2 and router 3 only (and not in router 1).
3. **Complete LI Aggregation**: LI aggregation performed at all routers.

Consider cases 2 and 3. Assume one receiver X has a fairly high loss rate, and all others have very low loss rates. Ideally, the number of LEs reaching through the source (after filtering) should be equal to the number of LEs generated by worst-loss receiver X. If a larger fraction of the LIs sent by X are suppressed by those sent from other low-loss receivers, then fewer and fewer LIs generated by X might reach the source. We call this the partial aggregation case (case 2). This could occur because X has a longer RTT, and as the number of receivers increase, we expect more overlaps between sequence numbers lost by X and other receivers. In the worst case, no LIs generated by X would reach the source. Then even after LI2LE filtering, the maximum number of LEs
Comparision of No. of Le's passed under different distributions of receivers under aggregators.

One receiver has 4% loss rate, all others 2% 6,000 packets transmitted

Figure 17: RATE AND RTT GRAPHS FOR TOPOLOGY WITH DIFFERENT BUFFERS (LOSS RATE AND RTTS) [WITH AGGREGATION]

Topology: Multilevel Tree with LI Aggregation

Source

Router 1

x% Receivers

Router 2

y% Receivers

Router 3

Figure 18: TOPOLOGY TO ILLUSTRATE LI AGGREGATION EFFECTS

generated by any single receiver would now become much lower, and thus affects the critical ratio $\frac{\max_i \lambda_i}{\sum_i \lambda_i}$ used by MaxLPRF. Hence MaxLPRF would pass through lesser number of LEs. The source hence would make fewer rate reductions, becoming more aggressive compared to TCP flows on the worst loss path. Observe that fewer rate reductions also means that the multicast flow even under cases of full (worst-case) aggregation does not experience the drop-to-zero problem.

Figure 19: PARTIAL AGGREGATION EFFECTS

The graphs shown in Figures 19 (a), (b), (c) illustrate the effects of no aggregation, partial and worst-case aggregation. The graphs plot the number of LEs passed by the filter cascade against the total number of receivers. The topology is same as in Figure 18. One receiver has loss probability of 4% (and has a longer RTT) while all others have 2%. This would bias the aggregation process against the worst-loss receiver's LIs. We vary the distribution of receivers under the two aggregators in the multicast tree. The notation $(x,y)$ in Figure 19 indicates
that $x\%$ of all the receivers (including the receiver with $4\%$ loss probability) are attached to Router 2 and remaining $y\%$ are attached to Router 3. The experiments were repeated for different random seeds and confidence intervals plotted. In each experiment, 6000 packets were sent which leads to an expectation of 240 LEs (given a worst-loss receiver having $4\%$ loss rate) after the cascade of filters is applied. If fewer than 240 LEs are passed, the multicast rate would be higher than competing TCP on the worst path.

In all graphs of Figures 19, the no-aggregation case passes just under 240 LEs (99% confidence intervals), which implies that the scheme works very well without aggregation. This is also similar to the results seen in Figure 12 where we used different loss probabilities and examined a larger number of receivers (up to 10,000 receivers). Now, the performance of the intermediate curve (partial aggregation) in Figures 19 varies depending upon the pattern of aggregation. Observe that if more LIs from the max loss receiver get through without begin aggregated, the intermediate (partial aggregation) curve is almost comparable to the scenario with no aggregation (eg: Figure 19 (a)). However, in the case of Figures 19 (b) & (c) the max loss receiver is present in the subtree with a larger number of receivers. Combined with the effect of longer RTT, these configurations bias the aggregation process against the worst-loss receiver because of the increased probability of overlap between the ranges of lost packets seen by the worst-loss receiver and other receivers connected to Router 2. Hence Figures 19 (b) & (c) shows the partial aggregation curve closer to the worst-case curve. The worst-case curve in all the three graphs depicts the effect of complete aggregation done at Router 1, which suppresses all the LIs from the worst-loss receiver.

In summary, aggregation poses a performance problem for LE-SBCC in the cases where the worst-loss receiver's feedback is significantly suppressed. However the effects lead to TCP unfriendliness only. Stability and drop-to-zero avoidance are solved. It should be noted that aggregation issues pose a problem for PGMC too (see [13]). Any randomization or technique to break the bias against such worst-loss receivers in the aggregation process (eg: see [13]) would solve this performance degradation issue both for LE-SBCC and PGMC.

4 TFRC module

TFRC [8] is a congestion control policy which provides smooth rate changes, which makes it suitable for audio/video applications. TFRC was originally designed for unicast. But our approach allows the use of the TFRC policy instead of the AIMD policy for rate adaptation. However, we need to modify TFRC to fit it into our framework.

4.1 TFRC Module Design

For AIMD, we used loss events (LEs) to trigger rate reductions. However, for TFRC we need to convert them into a LE rate to update the data transfer rate. TFRC [8] requires the receiver to calculate the LE rate and send it back to the sender. In contrast, our scheme is purely source based. There is no such kind of feedback from the receivers. We modify this mechanism so that the LE rate can be calculated at the source. Recall that the output of cascaded filters (Section 2.1) are loss events (LEs). The source can count the loss intervals (the number of packets between consecutive LEs) and calculate the LE rate and therefore the data transfer rate. The details follow.

With the following definitions,

- $s_0$: the interval since the most recent LE.
- $s_i$: the $i$-th most recent loss interval. ($i \geq 1$)
- $N$: the total number of loss intervals from the beginning of the data transfer.
- $n$: an integer greater than 0.
- $\alpha$: $0 \leq \alpha \leq 1$.

the average loss interval $\bar{s}_i$ is calculated as following.

$$\bar{s}_i = \frac{\sum_{k=1}^{n} s_{i+k-1} w_k + \sum_{k=i+n}^{N} s_k \alpha^{k-i-n+1} \sum_{k=1}^{n} w_k + \sum_{k=i+n}^{N} \alpha^{k-i-n+1} }{\sum_{k=1}^{n} w_k + \sum_{k=i+n}^{N} \alpha^{k-i-n+1} }$$
In [8], \( n = 8, \alpha = 0, w_1 \) to \( w_8 \) have values of 1, 1, 1, 0.8, 0.6, 0.4, 0.2 respectively. In our scheme, the randomness of the MaxLPF filter can cause larger variance of the average loss interval and therefore more oscillation of the LE rate. To counter this effect, we chose different values for these parameters. In our experiments, \( n = 8, w_1 \) to \( w_8 \) are all 1, and \( \alpha = 0.95 \). (When there are not enough loss intervals, (1) \( N < n \), set \( w_{N+1}, \ldots, w_n \) and \( \alpha \) to 0, (2) \( N = n \), set \( \alpha \) to 0.) They yield satisfying results. Note that this method and the parameter values need not be the only viable set for calculating average loss interval. Other methods and parameter values could be explored.

Whenever a LE is detected, \( \delta_1 \) is calculated, the reported LE rate \( p \) is \( 1/\delta_1 \). Then the data transfer rate \( T \) (bytes/sec) is calculated with the following formula [8] and enforced.

\[
T = \frac{\delta}{R \sqrt{\frac{2p}{s}} + t_{RTO} (3 \sqrt{\frac{2p}{s}}) p(1 + 32p^2)}
\]

where \( s \) is the packet size, \( R \) is round-trip time, \( p \) is LE rate, \( t_{RTO} = 3RTT + 4 \sigma \).

During absence of congestion, after each RTT, \( \delta_0 \) is re-calculated. The final average loss interval \( \delta \) is \( \max(\delta_0, \delta_1) \). If it is different from the previous value of \( \delta \), the rate \( T \) will be updated with the reported LE rate \( p = 1/\delta \). If congestion is absent for a long time, \( \delta_0 \) will become larger and larger, so is \( \delta \), and the reported LE rate will drop. Consequently, the rate \( T \) will increase, given that the RTT does not vary much. Note that our adaptation of TFRC for multicast differs from the recent work in TFMCC [12] where each receiver measures the RTT and loss event (LE) rate and the source has a filter to decide which feedback to use.

### 4.2 Simulation Results

The results presented here show that the LE-SBCC scheme with the TFRC module works relatively well.

![Figure 20: TFRC MODULE SIMULATION RESULTS](image)

(a) 1 RM vs 16 TCP Reno (instant rates)  (b) 1 RM vs 16 TCP Reno (avg. rates)

We tested the scheme with the TFRC module on the topology shown in Figure 5. The instant rates of multicast flow and two randomly selected TCP flows are shown in Figure 20 (a). Compared to Figure 13 (b), the rate changes are much smoother. Furthermore, the multicast flow is still fair to other TCP flows, as shown in Figure 20 (b)\(^7\).

In a summary, TFRC as a module can be installed into our framework and work well, demonstrating the modularity of the scheme and its applicability to multimedia.

### 5 Linux Implementation and Experimentation

For any scheme to be deployable it needs to be implemented and tested in real-world networks. We implemented LE-SBCC on a real network to study implementation issues and performance under real-world conditions. Specifically, we tested TCP friendliness and drop-to-zero problems. Lot of real world issues and limitations not found in simulation had to be tackled. Such issues are discussed in detail in the following sections.

\(^7\)avg. rate at time \( t \) = amount of data sent in period \( [0, t] / t \)
5.1 Implementation

The scheme is implemented on a simplified version of PGM. However, it is not limited to PGM or just reliable multicast. It has been implemented on top of UDP using RedHat Linux 7.0. The following aspects of PGM have been implemented:

- ODATA (original data packet), RDATA (retransmitted data packet), NAK (negative acknowledgement packet), NCF (NAK confirmation packet) packet formats specified in PGM.
- Transmit window at source.
- Receive window at receiver.

By implementing the features mentioned above, we achieved a reliable transport layer. On the other hand, for the sake of simplicity, we omitted certain aspects of PGM:

- SPM (source path message) packets.
- Network elements support.

SPM packets are sent by source periodically to receivers. They carry information about transmit window which affects the generation of NAK’s. To compensate this, each packet from source carries the information about transmit window. Whenever a receiver gets a packet (ODATA, RDATA or NCF), it checks the sequence number continuity and sends NAK’s if needed.

The program runs as a user process without requiring any root privileges. As a result, the granularity of the timer used is 10 ms which is quite coarse. Coarse timers can introduce unnecessary delays. Assume that an event is supposed to occur at time $t + \delta t$. The timer for it is started at $t$. If the $\delta t$ is less than 10 ms, due to the coarseness of the timer, the event will actually occur at $t + 10$ ms. Then extra delay of $10\text{ms} - \delta t$ is introduced. Notice that the extra delay can accumulate. That is, the extra delay introduced by the first event can be added to that introduced by the second event. To reduce such negative effects, the following mechanisms are used:

- After processing any timeout event, the next event is checked against the current time obtained with the API `gettimeofday()`. If the timeout should actually happen, it will be processed. This process continues until no more timeout events are available. Since times returned by `gettimeofday()` have microsecond resolution, the interval between two successive events can be less than 10 ms, provided events are processed sufficiently fast.

- When a timeout event is scheduled, its timeout value is checked. If its timeout is within 5 ms (half of the timer granularity) from now, it is immediately triggered. This introduces a minor random factor (< 5ms) into event scheduling, but reduces the unnecessary accumulative delays mentioned above. In fact, together with the first mechanism, this results in possible bursty events. When the events are for packet transmission, bursty packet transmission occur. The optimal solution for this problem would be to get a system timer of finer granularity.

- We introduce a variant of TBF to control the packet transmission rate at source. Upon arrival of a token, if there are enough packets, the source will transmit as many packets as needed to maintain the required transmission rate. If there are not enough packets, all of them would be transmitted.

By inspecting the time out information, we have observed that the mechanisms described above worked as expected, although the approximation leads to somewhat bursty transmissions.

5.2 Experimentation

The experiments were conducted on our testbed consisting of RedHat Linux 7.0 systems. During the experiments, the diff-serv package `tc` in RedHat Linux 7.0 were used to restrict bandwidth and link latency. Also, a multicast router daemon `mrouted`\(^8\) was used to route multicast packets from one LAN to others.

\(^8\)Source code and binaries of `mrouted` can be found online at [http://www.vesa.video.ja.net/mrcv](http://www.vesa.video.ja.net/mrcv)
The LE-SBCC program runs in user mode subject to OS priority and timer constraints. It cannot send packets at high rates (> 1Mbps). Furthermore, in real experiments, it is very difficult to strictly control certain parameters such as link latency. Thus it is not possible to match the simulations exactly. However, we show that our scheme is TCP friendly and avoids drop-to-zero in real networks.

5.2.1 TCP Friendliness Experiment

During the TCP friendliness experiment, TCP SACK was enabled. We used TBF queue discipline of a software package tc to restrict the bottleneck bandwidth. Details of the configuration are listed as follows:

- Round trip link latency (set by tc): 500 ms
- Bottleneck bandwidth (set by tc): 1Mbps
- Source initial transmission rate: 200Kbps
- Source initial RTT estimate: 200 ms
- Receiver maximum random backoff before sending NAK’s: 100 ms

The fixed part of RTT is the round trip link latency of 500 ms. Therefore, any RTT sample is at least of that value. Other parts of delay include hardware I/O delay, router buffer delay, OS processing time and so on. It is very hard to find out the exact delay.

Upon detecting a packet loss, a receiver waits for a certain amount of time before sending a NAK. This time is called backoff time in PGM. A backoff time is chosen randomly from the interval [0, max random backoff] according to uniform distribution. This backoff increases the RTT. The maximum random backoff time is a configurable parameter. When it is large in relation to RTT, it will affect RTT estimation. We chose this time to be 100ms so that the receivers have sufficient time to listen for NCF after a loss detection. This helps to avoid duplicate NAK’s for the same sequence number from different receivers. At the same time, this does not affect RTT significantly.

For this experiment, we used the topology in simulation (Figure 5) with the bandwidths and delays changed. All TCP sources and the multicast source ran on one machine. Behind the bottleneck, there were four receiver machines. To achieve load balancing, each of the receiver machine had four TCP receivers and one multicast receiver. The result, shown in Figure 21, roughly agrees with that of simulation.

In the graph, the average rate of multicast is higher than that of TCP. The reasons can be:

1. The approximation we introduced in the source rate-controller leads to somewhat bursty transmissions for multicast sources.
2. The implementation of our scheme is in user space, thus has lower priority and is less reactive to the network situation than TCP which runs in kernel and has higher priority.
3. The timer of our program is coarse, which could make our program less reactive.
4. TCP timeouts are triggered due to small window size, while the multicast scheme does not have such rate-cutting mechanisms.

The TCP flow in the graph was randomly chosen from 16 TCP flows. The results indicate TCP friendliness and drop-to-zero avoidance of our scheme.

6 Summary and Conclusions

We have presented an LE-oriented, purely source-based multi-sender multicast congestion control scheme (LE-SBCC) for reliable and unreliable multicast transport. Since the scheme is completely contained at the source, it is simple to implement and deploy.
The model is a simple cascade of filters followed by an AIMD module. The filters together transform the multicast tree to appear like a unicast path for the purposes of congestion control. In the section of theoretical analysis of loss events (LEs) probability versus loss indications (LIs) probability, we justified our choice of LEs for rate reduction consideration. By studying the effects of removing a filter from the cascade, we showed that all filters are important. Several simulation and Linux-based implementation experiments confirm the effectiveness of our scheme.

Without LI aggregation, the scheme correctly filters the LEs (converted from LIs): avoids the drop-to-zero problem while being TCP-friendly. With aggregation, the scheme is likely to be more aggressive towards TCP, but never incurs the drop-to-zero problem. Like all single-rate schemes, its scalability is limited by the heterogeneity of the tree which may depress the source rate.

In general, the AIMD module in our scheme may also be replaced by a source-based TFRC scheme [8] or rate-based binomial scheme [1]. With some modifications, we adapted the TFRC for unicast [8] to our framework. The performance shown by simulation experiments is satisfying. Since TFRC is suitable for audio/video transportation, our scheme can be applied to multimedia.

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References


APPENDIX

A  Pseudo Code
\textbf{event:} Send packet:
  Record $T_{send}[j]$ for seq $j$

\textbf{event:} LI[i][j] received for from receiver $i$ for seq $j$:
  call \texttt{estimateRTT()}
  call \texttt{LI2LEfilter()}
  if passed
    call \texttt{maxLPRFilter()}
    if passed
      call \texttt{ATFilter()}
      if passed
        rate = rate / 2
      endif
    endif
  endif
endif

\texttt{estimateRTT()} \{
  \texttt{RTT_{current} = T_{current} - T_{send}[j]}
  \texttt{\delta = SRTT - RTT_{current}}
  \texttt{SRTT = RTT_{current} + 0.125 \times \delta}
  \texttt{\sigma = \sigma + (0.125 \times (|\delta| - \sigma))}
\}\n
\texttt{LI2LEfilter()} \{
  if ($(t - T_{lastPass}[i]) > SRTT + 2\sigma$)
    $T_{lastPass}[i] = t$
    pass LI as LE
  else
    filter LI
  endif
\}

\texttt{maxLPRFilter()} \{
  update $X_i$, $\max X_i$, $\sum X_i$
  $P(accept) = \frac{\max X_i}{\sum X_i}$
  Accept LE w/ probability $P(accept)$
}\}

\texttt{ATFilter()} \{
  if (CongestionFlag == TRUE)
    filter LE
  else \{
    \texttt{SilenceFlag = TRUE}
    \texttt{SilencePeriodTimer = RTT / 2 + 2\sigma}
    \texttt{CongestionFlag = TRUE}
    \texttt{CongestionEpochTimer = SilencePeriod + SRTT + 4\sigma}
    Accept LE
  \}\}

\textbf{event:} CongestionEpochTimer expires:
  CongestionFlag = FALSE

\textbf{event:} Silence PeriodTimer expires:
  SilenceFlag = FALSE

\textbf{event:} RateIncreaseTimer expires:
  if (CongestionFlag == FALSE)
    rate += $\frac{MSS}{SRTT + 2\sigma}$
  else \{
    if (SilenceFlag == TRUE)
      no data transfer
    else
      no increase in rate
  \}\n  RateIncreaseTimer = RTT + 2\sigma