Rate-Based Flow Controllers in High Speed Networks: $\mathcal{H}^\infty$ Approach for the Single Bottleneck Case*

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Abstract

Explicit-rate feedback-based congestion control in high speed networks involve end-systems which send data at a rate determined by a distributed system of local flow controllers at bottleneck nodes. In this paper an $\mathcal{H}^\infty$ based controller is designed for rate feedback in a single bottleneck network, and it is tested with simulations under different operating conditions. The return trip time delays are taken into account, and their performance limitations are investigated. Stability robustness against uncertainties in time delays is also considered. Controller design specifications also include "fairness" to multiple users, "usage" optimization, and minimization of the transients in the queue length. Simulations are performed for the controller developed from $\mathcal{H}^\infty$ control theory, under varying time delays and saturation type nonlinearity in the system.

Keywords: Communication networks, Flow control, $\mathcal{H}^\infty$ control, Time delay systems.

1 Introduction

Congestion is the problem which occurs when demand for a resource outstrips the capacity. In computer networks, congestion may occur in routers and switches which are shared by several sources. Note that "flow control" is a term used for the end-to-end control of a single flow (or multiple flows on a per-flow basis). Congestion control has been studied widely in computer networks and communications literature mainly for high-speed data networks, [5, 21], and more recently for Asynchronous Transfer Mode (ATM) networks, [6, 13, 18, 22, 25].

There are two popular methods of controlling congestion involving feedback control. The "rate-based" method is the one where sources send data at a particular rate and adjust their rates based upon network feedback. The "window based" control (or "credit based" control) has the sources sending no more than a window of unacknowledged packets; sources adjust window sizes based on network feedback. The TCP congestion control algorithm [11] employs a window-based algorithm

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which probes for the optimal window through a combination of multiplicative and linear increase, and a multiplicative decrease policy. Packet loss (detected through a timeout or duplicate acknowledgements) is used as an implicit indication of congestion.

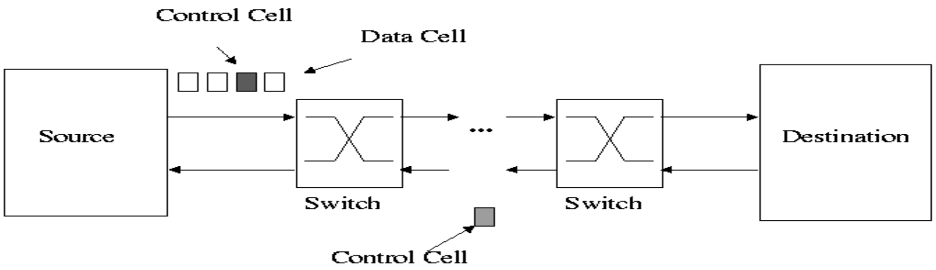

Figure 1: Control cell path

The Available Bit Rate (ABR) Service in ATM networks uses an explicit rate feedback-based, end-to-end traffic management framework [1]. This framework has three main components: source end-systems, switches (or network nodes) and destination end-systems. Once every N packets (called "cells") the sources send a control cell which can be used by switches to convey feedback. The control cells travel to the destination and are returned to the source in the same path as shown in Figure 1. Feedback signal may be in the form of a single bit or an explicit rate value, and can be written in the forward or reverse direction of travel of the control cell. In this framework the key elements of congestion control in a network are flow controllers at the bottleneck nodes, that determine the feedback signal. Indeed many of the papers cited above deal with this basic problem, see for example [5, 12, 18, 22, 25] and their references. For a survey of different traffic management schemes in ATM networks see [15]; for details and simulations of a specific empirically-derived scheme (ERICA) we refer to [14]. This approach may also be applied in multi-protocol label switching (MPLS) which is an internetworking technology between IP and ATM [4, 7].

In this paper we design a control-theoretic solution to the following problem involving a single bottleneck which receives data from n sources. The data flow rates from these sources are assumed to be generated by a feedback controller. Due to physical restrictions there are time delays in data flow between the sources and the bottleneck node. A feedback controller is to be designed for this time delay system. There are several controller design methods for different classes of systems with time delays, see e.g. [16, 20, 23] and their references. The techniques developed in [10, 24] are used in this paper. One of the design goals considered here is "fairness" to multiple users trying to send information through the same bottleneck node. That is, the minimum rate allocated to individual sources should be maximized. When sources are unconstrained (i.e., the demand is always greater or equal to the allocation), this reduces to simply equalizing the rates of sources. Another design objective is to maximize the "utilization," i.e. total data flow into the node should be kept close to the full "capacity" (maximum allowable flow rate for the data leaving the node) and the queue size should be kept close to a certain desired size. However, the most important and distinguishing design specification we consider is *stability robustness* with respect to uncertainties in the values of time delays in each flow path.

The paper is organized as follows. In Section 2, a mathematical model of the system is given. In Section 3, an $\mathcal{H}^\infty$ optimization problem is posed to address various issues related to feedback controller design. In Section 4, a computational procedure is outlined for the optimal controller. Stability robustness margins are derived numerically in Section 5. Controller implementation and modifications of the linear $\mathcal{H}^\infty$ optimal controller are discussed in Section 6, and time domain simulation results are presented. Concluding remarks are made in the last section.

2 Mathematical Model

We consider a bottleneck node with n source connections. Let $q(t) \geq 0$ denote the queue size at the bottleneck node, and $r_i(t) \geq 0$ be the flow rate at the ith source. The maximum rate at which the ith source can send data is denoted by d_i, i.e. $r_i(t) \leq d_i$. The rates $r_1(t), \ldots, r_n(t)$ are assigned to the sources by a feedback controller which measures the queue, $q(t)$, at the bottleneck node. The capacity is the outgoing flow rate, and it is denoted by $c(t)$. See Figure 2.

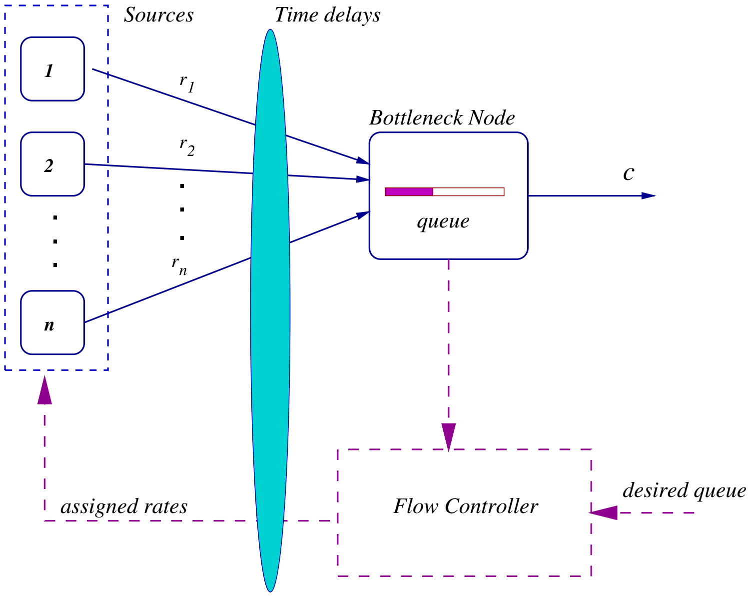

Figure 2: Flow Control Problem

For the $\mathcal{H}^\infty$ controller design the system is assumed to be linear time invariant. The main problem to be addressed is this: Given nominal values for time delays, and upper bounds on delay uncertainty, design a controller $K(s)$ such that closed loop system is robustly stable. It is also desired to keep $q(t)$ close to $q_d(t)$, which is usually a step-like function.

A simple dynamical model of this system is

$$\dot{q}(t) = \sum_{i=1}^{n} r_i(t-\tau_i) - c(t) \tag{1}$$

where τ_i is the return trip time delay between the ith source and the bottleneck node. The utilization

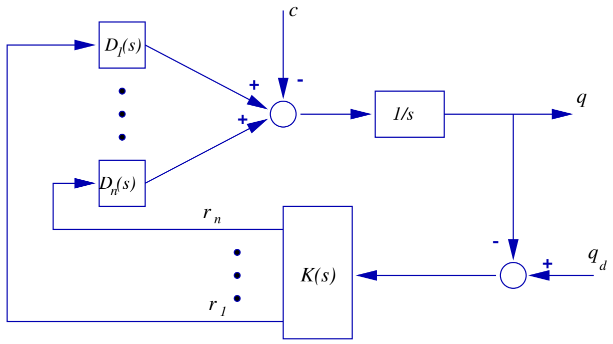

Figure 3: Feedback control system with MISO plant.

is defined to be the quantity

$$\vartheta(t) := \min\left\{ \frac{\sum_{i=1}^{n} r_i(t-\tau_i)}{c(t)} \; , \; 1 \right\}.$$

One of the design objectives is to have $\vartheta(t) \approx 1$. The controller should also achieve "fairness" in steady state, i.e. the rates allocated to different sources should be as close to each other as possible. A trivial choice is then to select equal rates for the sources, with a steady state value c_∞/n, where $c_\infty = \lim_{t\to\infty} c(t)$. In practice the flow rates are restricted by $0 \le c(t) \le c^+$, and $0 \le r_i(t) \le d_i$, for all $i = 1, \dots, n$, for some upper bounds c^+ and $d_1, \dots, d_n$. This nonlinearity will be ignored at the $\mathcal{H}^\infty$ controller design stage, but it will be taken into account when the resulting controller is modified, and the feedback system is simulated under different conditions.

In the frequency domain, the feedback system can be represented by transfer functions as shown in Figure 3, where q_d represents the "desired" queue length at the bottleneck node, $K(s)$ is the controller and $D_i(s) = e^{-\tau_i s}$ is the time delay in the ith channel.

The plant is a Multi Input Single Output (MISO) system whose transfer function is

$$P(s) = \frac{1}{s}[D_1(s) \; , \; \dots \; , \; D_n(s)].$$

The controller is a SIMO system with transfer function $K(s) = [K_1(s) \; , \; \dots \; , \; K_n(s)]^{\mathsf{T}}$. In general time delays $\tau_1, \dots, \tau_n$ are unknown (in fact they might be time varying; here they are assumed to be fixed). Let $h_i \ge 0$ be a given nominal value of τ_i, and define $\delta_i := (\tau_i - h_i)$. Let $\delta_i^+ \ge 0$ be a known estimate of the size of the uncertainty, i.e. $\delta_i \in (-\delta_i^+ \; , \; \delta_i^+)$.

In the next section the model developed here will be considered for feedback control. It should be mentioned that several different control schemes have been determined in [2, 3, 18, 22, 25] for similar mathematical models in discrete-time as well as continuous-time. In this paper robustness to uncertainty in the time delays will be taken into account by putting the problem into the framework of $\mathcal{H}^\infty$ control.

3 Feedback Controller Design

3.1 Robust stability

The following is a coprime factorization in $\mathcal{H}^\infty$ for the nominal plant

$$P_o(s) = \frac{1}{s}\,[e^{-h_1 s}\ ,\ \dots\ ,\ e^{-h_n s}] = N_o(s)M(s)^{-1}$$

where $M(s) = \frac{s}{s+\varepsilon} I_{n\times n}$ and $N_o(s) = \frac{1}{s+\varepsilon}\,[e^{-h_1 s}\ ,\ \dots\ ,\ e^{-h_n s}]$, $0 < \varepsilon$ is arbitrary. The uncertain plant can be represented as $P = (N_o + \Delta_N)M^{-1}$ with

$$\Delta_N(s) = \frac{1}{s+\varepsilon}\,[(e^{-\delta_1 s} - 1)\ ,\ \dots\ ,\ (e^{-\delta_n s} - 1)]\ \mathrm{diag}(e^{-h_1 s}, \dots, e^{-h_n s}).$$

Using standard results of robust control theory (see e.g. [26]) it can be shown that a controller $K(s)$ is robustly stabilizing (i.e. it stabilizes all plants of the form $P(s)$) if K stabilizes P_o, and

$$\|WK(1 + P_o K)^{-1}\|_\infty \le 1, \tag{2}$$

where $W = [w_1, \dots, w_n]$, with $w_i, w_i^{-1} \in \mathcal{H}^\infty$ and

$$|w_i(j\omega)| > \left|\frac{e^{-j\delta_i\omega} - 1}{j\omega}\right| \qquad \forall\ \omega \in \mathbb{R},\ \ \delta_i \in (-\delta_i^+\ ,\ \delta_i^+).$$

A slightly conservative, yet simple, choice is $w_i(s) = \delta_i^+$.

3.2 Performance issues

One of the performance objectives of rate-based congestion control is to keep the queue size, $q(t)$, as close to its desired value, $q_d(t)$, as possible. Consider $q_d(s) = \frac{1}{s}\widehat{q}(s)$ where $\widehat{q}$ is an arbitrary bounded energy signal. For example if $\widehat{q}$ is a pulse of finite duration, then q_d is a saturating ramp. The tracking error, $e(t) = q_d(t) - q(t)$, satisfies the following frequency domain identity:

$$e(s) = S_o(s)\ \frac{1}{s}\ (\widehat{q}(s) + c(s)), \qquad \text{where} \qquad S_o(s) = (1 + P_o(s)K(s))^{-1}.$$

Since K is a stabilizing controller, and P_o has a pole at $s = 0$, we have that $S_o(0) = 0$, and by the final value theorem the steady state value of the error $e_{ss} = \lim_{s\to 0} sE(s)$ is

$$e_{ss} = \lim_{s\to 0}([e^{-h_1 s}, \dots, e^{-h_n s}]K(s))^{-1} s(\widehat{q}(s) + c(s)) = ([1, \dots, 1]K(0))^{-1} c_\infty,$$

(note that the signal $\widehat{q}(t)$ is assumed to have finite energy, so its final value is zero). Thus, at least one of the entries of the controller must have a pole at $s = 0$, in order to have zero steady state error. The above analysis remains valid when $P_o(s)$ is replaced by $P(s)$. So, the steady state is zero not just for the nominal system but for the actual system, provided the closed loop system is robustly stable with a controller having a pole at $s = 0$.

Also note that the rate feedback signals are given by

$$[\, r_1(s) \quad \cdots \quad r_n(s) \,]^{\mathrm{T}} = \frac{K(s)}{1 + P_o(s)K(s)} \; \frac{1}{s} \; (\widehat{q}(s) + c(s)), \tag{3}$$

and for "fairness" it is desired to have the steady-state values of $r_1, \ldots, r_n$ equal to c_∞/n. This means that the entries, $K_i(s)$, of the controller $K(s)$ must satisfy

$$\lim_{s \to 0} K_i(s)([e^{-h_1 s}, \ldots, e^{-h_n s}]K(s))^{-1} = \frac{1}{n} \quad \text{for all } \; i. \tag{4}$$

A simplistic approach for flow control would be to select each $r_i(t) = c_\infty/n$; but this would lead to $q(t) = 0$, which is not necessarily the desired steady state value of the queue. Also, this simple control law would not compensate time variations in $c(t)$, and time delay uncertainty.

In summary, besides the steady-state behavior, it is also desired to control the transient response of the system. For this purpose the $\mathcal{H}^\infty$ norm of the weighted sensitivity function can be minimized (this corresponds to worst energy minimization for the tracking error, see for example [8, 10]). More precisely, the problem is to

$$\text{minimize} \quad \|W_s(1 + P_o K)^{-1}\|_\infty \tag{5}$$

over all controllers $K(s)$ stabilizing $P_o(s)$, where $W_s(s)$ is the sensitivity weighting filter.

3.3 An $\mathcal{H}^\infty$ optimization problem

For the solution of (5) under the condition (2) there is significant amount of degree of freedom in K. Time delays in the feedback paths between different sources and the bottleneck node can be equalized by the controller by choosing

$$K(s) = D_e(s) \frac{1}{nh} K_\nu(s) \tag{6}$$

where $K_\nu(s)$ is the scalar part of the controller (to be specified later),

$$D_e(s) = [e^{-(h-h_1)s}, \ldots, e^{-(h-h_n)s}]^{\mathrm{T}}, \quad \text{and} \quad h := \max\{h_1, \ldots h_n\}.$$

The purpose of $D_e(s)$ is to equalize the nominal time delays in communication channels. This structure of $K(s)$ satisfies (4), and furthermore it simplifies the design considerably.

When the controller is given by (6), the robustness condition (2) is equivalent to

$$\|W_\nu K_\nu(1 + P_\nu K_\nu)^{-1}\|_\infty \leq 1 \tag{7}$$

where $P_\nu(s) = e^{-hs}/hs$, and $W_\nu(s)$ is a new uncertainty weight determined from

$$|W_\nu(j\omega)| \geq \frac{1}{nh} \left| \sum_{i=1}^{n} \delta_i^+ e^{-j(h-h_i)\omega} \right| \qquad \forall \; \omega. \tag{8}$$

A simple choice is $W_\nu(s) = \rho$ where

$$\rho := \frac{1}{nh}\sum_{i=1}^{n} \delta_i^+ =: \frac{\delta}{h} \ . \tag{9}$$

Considering the performance problem (5) we can now and define a 2-block optimization problem: minimize $\gamma > 0$ such that K_ν is stabilizing P_ν and

$$\left\| \begin{bmatrix} W_s(1+P_\nu K_\nu)^{-1} \\ W_\nu K_\nu(1+P_\nu K_\nu)^{-1} \end{bmatrix} \right\|_\infty \le \gamma. \tag{10}$$

This is a two block $\mathcal{H}^\infty$ problem for the SISO plant $P_\nu(s) := e^{-hs}/hs$.

4 Computation of the $\mathcal{H}^\infty$ optimal controller

A careful study of the $\mathcal{H}^\infty$ controller formula given in [24] shows that the poles of the sensitivity weight, $W_s(s)$, appear as poles of the open loop gain $L_\nu(s) := P_\nu(s)K_\nu(s)$. In Section 3.2 it is determined that the controller $K_\nu(s)$ must have a pole at $s = 0$ for good steady state response. Therefore, since $P_\nu(s)$ has a pole at $s = 0$, $W_s(s)$ must have double poles at $s = 0$, for example

$$W_s(s) = \frac{1}{(hs)^2}. \tag{11}$$

Now define $W_1(s) = W_s(s)$, and $W_2(s) = \rho hs$, where $\rho = \delta/h$ is the relative delay uncertainty. Then, the problem (10) is equivalent to the following mixed sensitivity minimization problem:

$$\gamma_{\rm opt} := \inf_{K_\nu \ stabilizing \ P_\nu} \left\| \begin{bmatrix} W_1(1+P_\nu K_\nu)^{-1} \\ W_2 P_\nu K_\nu(1+P_\nu K_\nu)^{-1} \end{bmatrix} \right\|_\infty . \tag{12}$$

The fact that $|e^{-jh\omega}| = 1$ is used in transforming the problem (10) into (12). The problem data in (12) is a function of the normalized frequency $\hat{s} := hs$, so will be the optimal solution $K_{\nu,\rm opt}$.

By applying the formulae given in [24], the optimal scalar controller $K_{\nu,\rm opt}$ can be easily computed:

$$K_{\nu,\rm opt}(s) = \frac{R(\hat{s})}{1+R(\hat{s})\frac{e^{-\hat{s}}}{\hat{s}}} \left(\frac{1-\gamma_{\rm opt}^2 \hat{s}^4}{\gamma_{\rm opt}^2 \hat{s}^4} \right) \tag{13}$$

where $R(\hat{s})$ is in the generic form

$$R(\hat{s}) = \frac{\gamma_{\rm opt}}{\rho} \ \frac{\hat{s}^3(\alpha-\hat{s})}{(\hat{s}+a)(\hat{s}^2+b\hat{s}+c)(\alpha+\hat{s})}.$$

The optimal cost $\gamma_{\rm opt}$, and the parameters a, b, c, α are calculated as follows: for a fixed $\gamma > 0$ let x be the unique positive root of

$$x^3 + \frac{1}{\gamma^2}x^2 - \frac{1}{\rho^4} = 0$$

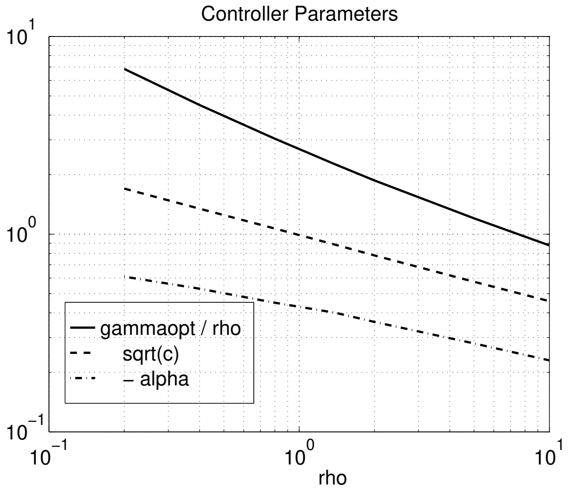

Figure 4: Controller Parameters

and define

$$a = \frac{1}{\rho\sqrt{x}}\ , \quad c = \rho a\ , \quad b = \sqrt{2c - a^2}\ , \quad \alpha = \frac{1}{\sqrt{\gamma}}\,\frac{1 - \rho e^{\frac{1}{\sqrt{\gamma}}}(\frac{1}{\sqrt{\gamma}} + a)(\frac{1}{\gamma} + b\frac{1}{\sqrt{\gamma}} + c)}{1 + \rho e^{\frac{1}{\sqrt{\gamma}}}(\frac{1}{\sqrt{\gamma}} + a)(\frac{1}{\gamma} + b\frac{1}{\sqrt{\gamma}} + c)}$$

then γ_{opt} is the largest γ which satisfies

$$1 + R(\beta)\frac{e^{-\beta}}{\beta} = 0, \qquad \text{with} \qquad \beta = j\frac{1}{\sqrt{\gamma}}.$$

The following table shows the parameters of the optimal controller, computed for different values of ρ.

ρ	0.2	0.8	2	10
γ_{opt}	1.37	2.43	3.74	8.77
$-\alpha$	0.61	0.45	0.36	0.23
a	1.73	1.10	0.82	0.49
b	1.67	1.03	0.75	0.42
$\sqrt{c}$	1.70	1.07	0.78	0.46

The controller is determined by the parameters of $R(\hat{s})$. The numerical results show that three stable poles of $R(\hat{s})$ have approximately the same magnitude since $a \approx \sqrt{c}$. Moreover, the choice of ρ does not seem to affect the phase of the complex poles: their phase is approximately constant at $\pm 130°$. See Figure 4 for the plots of ρ versus the gain $\gamma_{\mathrm{opt}}/\rho$, magnitude of stable poles $a \approx \sqrt{c}$, and the unstable pole, $-\alpha$.

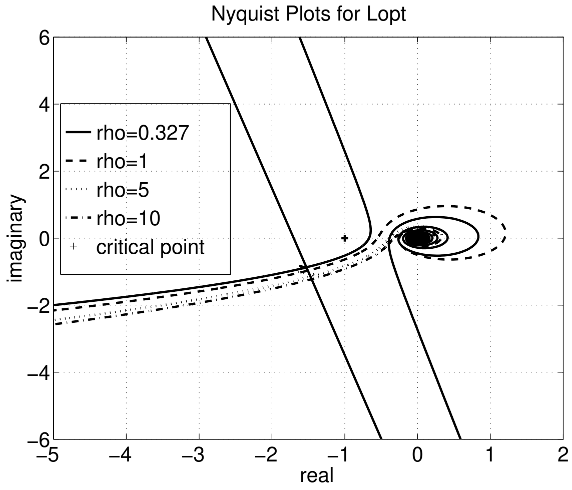

Figure 5: Nyquist Plots.

5 Stability Robustness Margins

An interesting point to note is that the optimal open loop gain is

$$L_{opt}(s) = P_\nu(s)K_{\nu,\mathrm{opt}}(s) = P_o(s)K_{\mathrm{opt}}(s) = \frac{R(\hat{s})\frac{e^{-\hat{s}}}{\hat{s}}}{1+R(\hat{s})\frac{e^{-\hat{s}}}{\hat{s}}}\left(\frac{1-\gamma_{\mathrm{opt}}^2\hat{s}^4}{\gamma_{\mathrm{opt}}^2\hat{s}^4}\right), \tag{14}$$

which is a function of the normalized frequency $\hat{s}$. This means that in the $\mathcal{H}^\infty$ optimal design setting considered here the bandwidth of the optimal system is inversely proportional to the time delay h. The Nyquist plots for L_{opt} are shown in Figure 5 for different values of ρ. Since the plant P_ν does not have any poles in the open right half plane, and since the controller stabilizes the feedback system by design, it can be deduced that the optimal controller is unstable for $\rho \leq 0.327$. The smallest value of ρ that yields a stable optimal controller is calculated to be approximately 0.33. For such time delay systems it was shown that as ρ decreases the number of encirclements of the critical point (-1,0) increases, and in the limit for $\rho = 0$ we expect to have a controller with infinitely many unstable poles, [9, 17].

Note that for good stability robustness (i.e. $L_{\mathrm{opt}}(j\omega)$ is far from the critical point) the design parameter ρ should be large. On the other hand, if ρ is too large, time domain response can be too slow; this point will be demonstrated later when controller implementation and simulations are discussed. The minimum distance between $(-1,0)$ and $L_{\mathrm{opt}}(j\omega)$ is

$$\eta := \inf_\omega |1 + L_{\mathrm{opt}}(j\omega)| = \left(\|(1+P_oK_{\mathrm{opt}})^{-1}\|_\infty\right)^{-1}$$

The plot of ρ versus η is shown in Figure 6.

By the definition of two block problem (12)

$$\|\frac{\rho}{\gamma_{\mathrm{opt}}}K_{\nu,\mathrm{opt}}(1+P_\nu K_{\nu,\mathrm{opt}})^{-1}\| \leq 1,$$

so a lower bound of the maximal allowable relative time delay uncertainty $(h_{\max} - h)/h$ is $\rho/\gamma_{\mathrm{opt}}$. It is well known that, for a stable feedback system with open loop transfer function $L_o(s)$, the delay

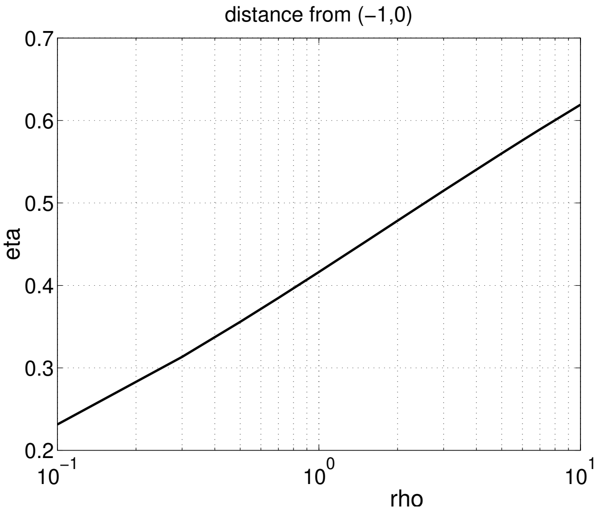

Figure 6: A Robustness Measure: η versus ρ.

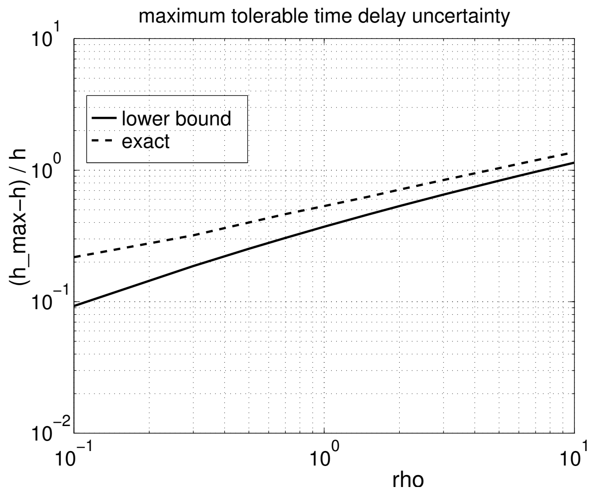

Figure 7: Largest Allowable Time Delay Uncertainty.

margin (i.e. the largest $h' > 0$ for which the feedback system remains stable when the open loop transfer function is perturbed to $e^{-h's}L_o(s)$) is given by $h' = \phi/\omega_c$, where ϕ is the phase margin in radians, and ω_c, in radians/sec., is the gain crossover frequency (i.e. $|L_o(j\omega_c)| = 1$) determined from the Bode or Nyquist plots of $L_o(s)$. Hence the quantity $(h_{\max} - h)/h$ can be determined exactly from $L_{\text{opt}}(j\omega)$; Figure 7 shows the lower bound and the exact value of the maximum allowable time delay uncertainty as a function of ρ.

6 Controller Implementation and Simulation Results

The controller expression given by (13) cannot be implemented in a stable manner, because there are unstable pole-zero cancellations internally in the controller: some of the the unstable roots of $(1 + R(\hat{s})\frac{e^{-\hat{s}}}{\hat{s}})$ are canceled by the roots of $(1 - \gamma_{\text{opt}}^2 \hat{s}^4)$. But a simple algebra shows that the optimal

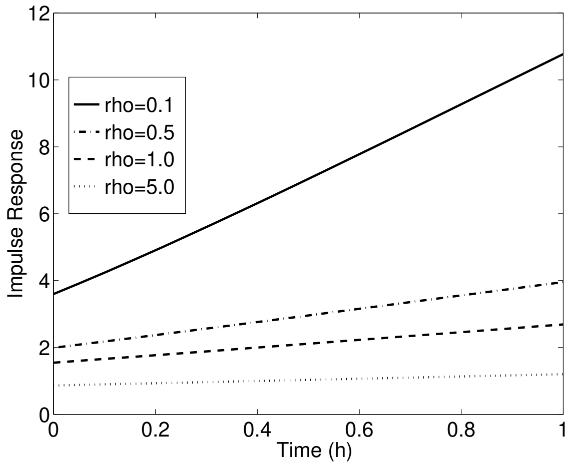

Figure 8: Impulse Response of $F(\hat{s})$.

controller (13) can be implemented in a stable fashion as follows:

$$K_{\nu,\mathrm{opt}} = \frac{\gamma_{\mathrm{opt}}}{\rho} \, \frac{(\hat{s}-\alpha)}{\hat{s}} \, \frac{1}{1+F(\hat{s})} \tag{15}$$

where

$$F(\hat{s}) = \frac{(\hat{s}+a)(\hat{s}^2+b\hat{s}+c)(\hat{s}+\alpha)-(\hat{s}^4-\gamma_{\mathrm{opt}}^{-2})}{(\hat{s}^4-\gamma_{\mathrm{opt}}^{-2})} - \frac{\gamma_{\mathrm{opt}}\rho^{-1}\hat{s}^2(\hat{s}-\alpha)e^{-\hat{s}}}{(\hat{s}^4-\gamma_{\mathrm{opt}}^{-2})}. \tag{16}$$

The interpolation conditions from which controller parameters $\{\gamma_{\mathrm{opt}}, \alpha, a, b, c\}$ are determined, [24], imply that $F(\hat{s})$ and $e^{-\hat{s}}F(-\hat{s})$ are stable. Therefore all of the four poles of $F(\hat{s})$ are canceled internally in (16). In terms of state space realizations of the two additive terms of (16), a realization of $F(\hat{s})$ can be given in the form

$$F(\hat{s}) = C(e^{-\hat{s}}I - e^{-A})(\hat{s}I - A)^{-1}B \tag{17}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \gamma_{\mathrm{opt}}^{-2} & 0 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad C = \frac{\gamma_{\mathrm{opt}}}{\rho} \, [\, 0 \quad 0 \quad \alpha \quad -1 \,] \, .$$

The expression (17) demonstrates that the impulse response of $F(\hat{s})$ is restricted to the time interval $[0\ ,\ 1]$. A similar $\mathcal{H}^\infty$ controller structure was obtained in [19] for delay systems. The impulse response of $F(\hat{s})$ is shown in Figure 8 for several different values of ρ.

A digital implementation of the controller (15) has a PI term cascaded with a feedback block containing an FIR filter. The length of the FIR filter is h/T_s, where T_s is the sampling period. If h is large with respect to T_s then the order of the FIR filter, hence the order of the controller is large.

The controller (15) is derived from $\mathcal{H}^\infty$ optimization, so nonlinearities in the actual system are not taken into account. Recall that the maximal data rate (demand) for the ith source is d_i, and in

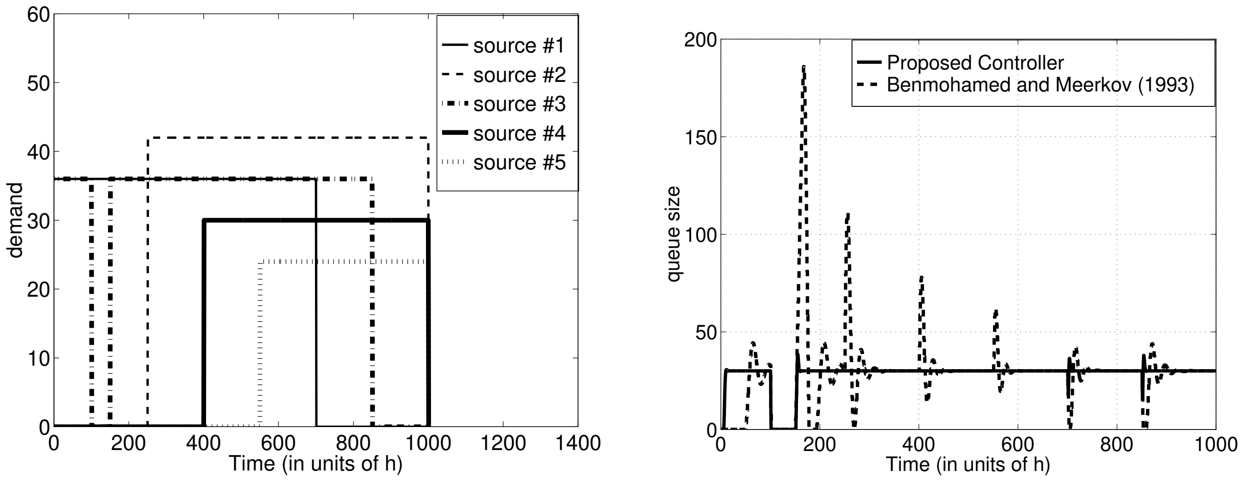

Figure 9: Source Connection Times and Demands, and Resulting Queue Size.

steady state the controller assigns equal rates, c_∞/n, to different sources. If the assigned rate is higher than d_i for the ith source, then the rate command for this particular source should be saturated at d_i. Suppose that $(d_1(t) + \ldots + d_n(t)) \geq c(t)$ for all t, i.e. total demand is higher than the capacity of the bottleneck link. In this case, although some sources may be saturated, by distributing the remaining total rate allocation to sources with higher demands, overall operation can be kept at the linear range. Another key assumption made in the controller design is that the number of active sources, n, and corresponding d_i's are known. This is a reasonable assumption in the sense that when sources sign-in and sign-off from the network this type of information is made available to the flow controller. Accordingly, in the SIMULINK based implementations modifications are made on the basic linear controller (15) to include logical blocks to perform saturation checks, rate allocations to different sources, time delay equalization units, and a counter for the number of active sources.

In order to reduce the effect of the saturation type of nonlinerity in the queue size, the integrator in (15) is implemented with an anti-windup logic. In the SIMULINK based implementation used for time domain results presented here, the integration, at the PI term of the controller, is stopped if all sources become saturated, and then the integrator is reset to zero when at least one of the sources becomes unsaturated; that prevents excessive overshoot in the queue size.

The benchmark example selected for simulations is from [5], where five different sources are sending data through a bottleneck node at different time instants with different demands. Considering Table II of [5] the roundtrip delays for sources 1, 2, 3, 4, and 5 are taken to be 0, 0, $20\tau_s$, $40\tau_s$, and $60\tau_s$, respectively, where τ_s is the transmission time of a single packet. Thus, the maximum roundtrip delay is $h := 60\tau_s$. The computational time delays are neglected. Following the example given in [5] the queue clearing rate is chosen as $c(t) = 60$ packets per h unit of time, and desired queue is taken to be $q_d = 30$ packets. Figure 9 shows a comparison of the response corresponding to the controller proposed here (for $\rho = 2$) with the benchmark response given in [5]. The benchmark controller in [5] updates its output once every h units of time, whereas in the simulations performed for the controller developed here the controller output is updated 20 times per h units of time, i.e. $T_s = h/20$.

The step response ($q(t)$ versus t) is obtained by choosing a sufficiently large fixed value for the total demand. For example, if the simulation, shown in Figure 9, is stopped at time instant $30h$, then it simply gives the response to demands of first and third sources (total demand here is $(36 + 36) > 60 = c(t)$, and because of delay equalization for all five sources, the total time delay is still $h = 60\tau_s$).

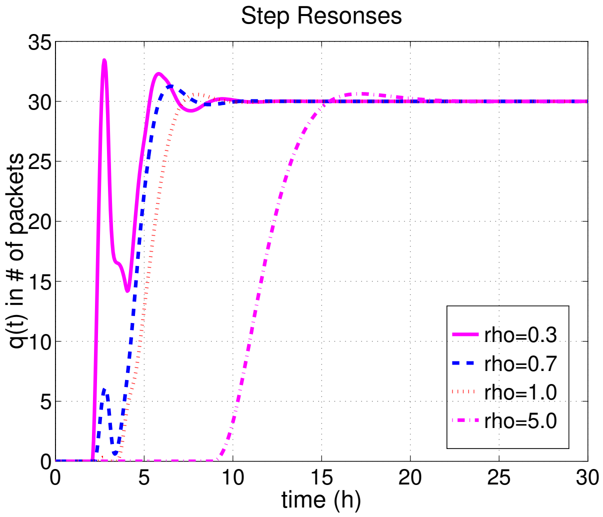

Figure 10: Step Responses.

For different values of ρ, the step response is as shown in Figure 10. Note that larger the value of ρ, slower the step response. The basic trade-off in selection of ρ is now clear: if ρ is too large the response is slow, but if it is too small the stability robustness margin is small. By looking at the step responses and the delay margin plots it is determined that $\rho = 2$ is a reasonably good choice.

Figure 11 shows the step responses corresponding to the cases where the controller uses h as the time delay, but actual time delay is different, say τ. It can be seen that the system enters into a limit cycle when actual time delay is $\tau = 2h$ i.e. it is 100% more than the value of the time delay used in the controller. This not surprising since for the value of $\rho = 2.0$ Figure 7 shows that the largest allowable time delay uncertainty is about 70%, i.e. the system can tolerate $\tau = 1.7h$ but $\tau = 2h$ leads to instability.

The step response corresponding to a time-varying delay uncertainty is shown in Figure 12. In this case all five sources are made active (the number of active sources detected by the controller as a function of time is also shown in the figure), and additive delay perturbations in the form $h(0.5 + \sin(\frac{2\pi}{10h}t))$ are injected to channels 4 and 5 (in these channels nominal time delays are $\frac{2}{3}h$ and h, respectively). The additive delay perturbations in channels 4 and 5 take values between $-0.5h$ and $1.5h$, with and average of $0.5h$. Note that the nominal delay in other channels are less than $0.5h$, to avoid negative time delays (non-causal situations) the time delay perturbations are not introduced in the first three channels.

The effect of variations in $c(t)$ is demonstrated in Figure 14, where $q_d(t) = 30$ packets is fixed and $c(t)$ is taken to be a time varying function shown in the Figure 13. It is observed that fast sinusoidal variations in $c(t)$ lead to relatively small variations in $q(t)$. The effects of sudden "step-like" changes are more significant, but they are transient.

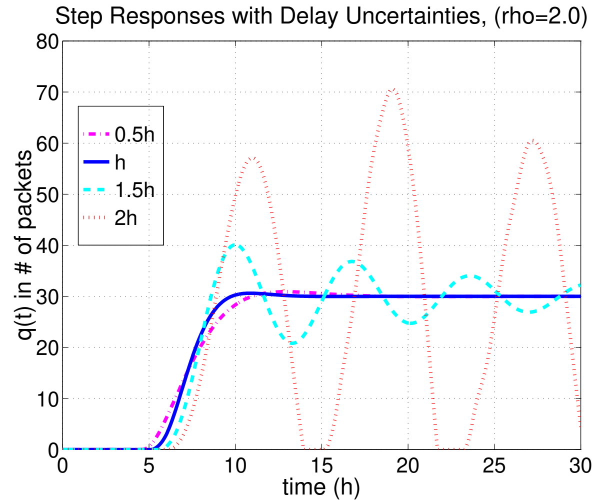

Figure 11: Step Response with Uncertain Delays, $\rho = 2.0$.

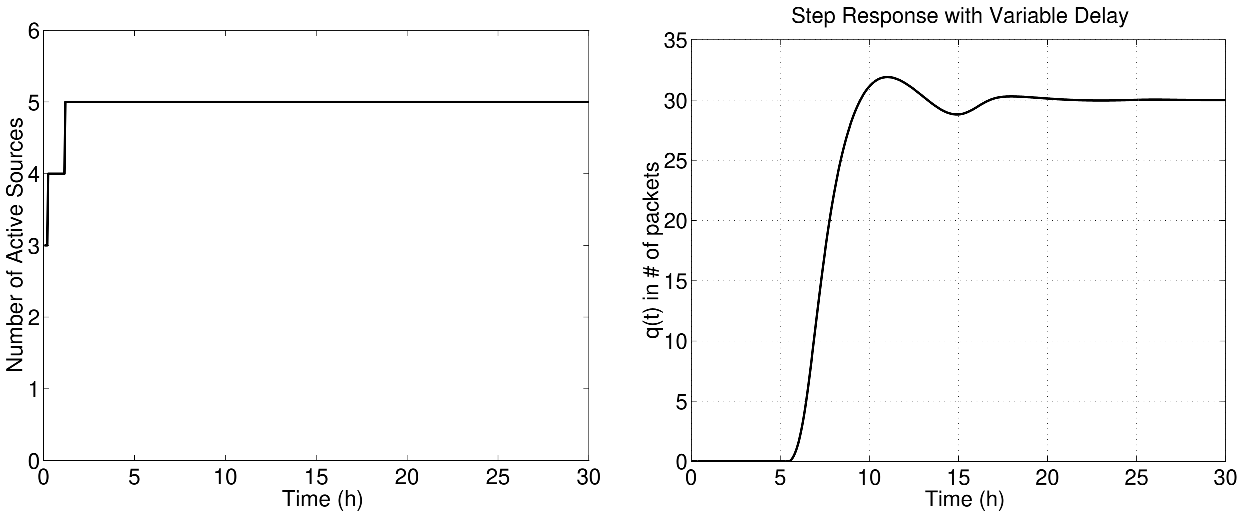

Figure 12: Step Response with Time Varying Delays, $\rho = 2.0$.

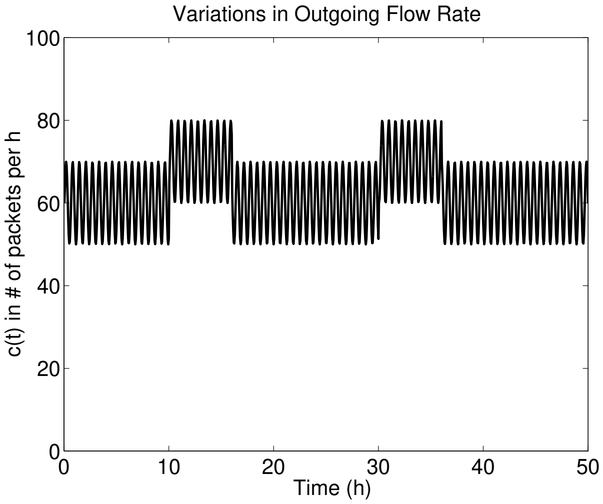

Figure 13: Variations in $c(t)$.

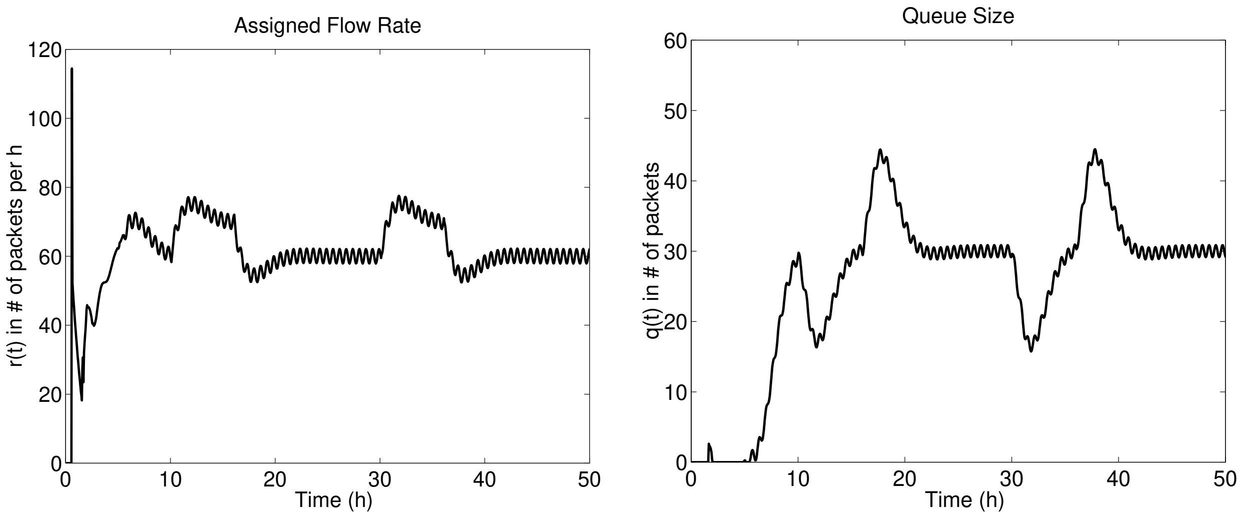

Figure 14: Response with Non-Constant $c(t)$, $\rho = 2.0$.

7 Conclusions

In this paper an $\mathcal{H}^\infty$ controller is designed for flow control in high speed networks with a single bottleneck node. The optimal controller is implemented in a stable fashion: in the form of a PI term cascaded with a feedback term involving a FIR filter. This controller robustly stabilizes the feedback system in the presence of delay uncertainties. It also achieves fairness in steady state, and maximizes utilization. The design framework considered here allows fluctuations in the capacity $c(t)$. Minor non-linear modifications are made in the SIMULINK based implementation in order to minimize the time interval in which the system is in saturation.

Future work includes extensions of this approach to multiple bottleneck case, and algorithmic implementation of the controller on a real ATM switch.

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